

**HANDWRITTEN
NOTES
OF**

(THEORY OF MACHINE)

BY

NewtonDesk.Com

Theory of M/c:

- Simple Mechanics] ✓
- Motion Analysis]
 - Velocity analysis
 - Acceleration Analysis
- Gears]
- Gear trains] ✓
- Governors ✓
- Flywheel ✓
- Balancing ✓
- Mechanical vibrations ✓
- Cams & followers ✓
- Gyro scope ✓

4 days

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Mechanical Engineering



Engineering of Mechanics



Study of Motion

← Kinematic

→ Dynamics (mass)

Study of Motion without
the consideration of Basic
Cause of Motion i.e force

$$\vec{v} = \frac{d\vec{s}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt}, \quad \vec{j} = \frac{d\vec{a}}{dt}$$

Study of Motion with
the consideration of the basic
Cause of motion i.e force.

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Kinematic Link or Element :-

"Every Part of a machine, which is having some relative motion with respect to some other part will be known as, 1 Kinematic Link or Element". e.g.:- Man standing on floor

It is necessary for the link, to be the resistant body so that it is capable of transmitting power and motion, from one element to the other element.

Types of Link :-

1) Rigid Link :-



Deformations are Negligible.

2) Flexible Link



Deformations are there, but are in permissible limits.

for e.g.:-

Belt, Rope, chain drives

for e.g.:-

Crank, Connecting Rod
Piston, Cylinder

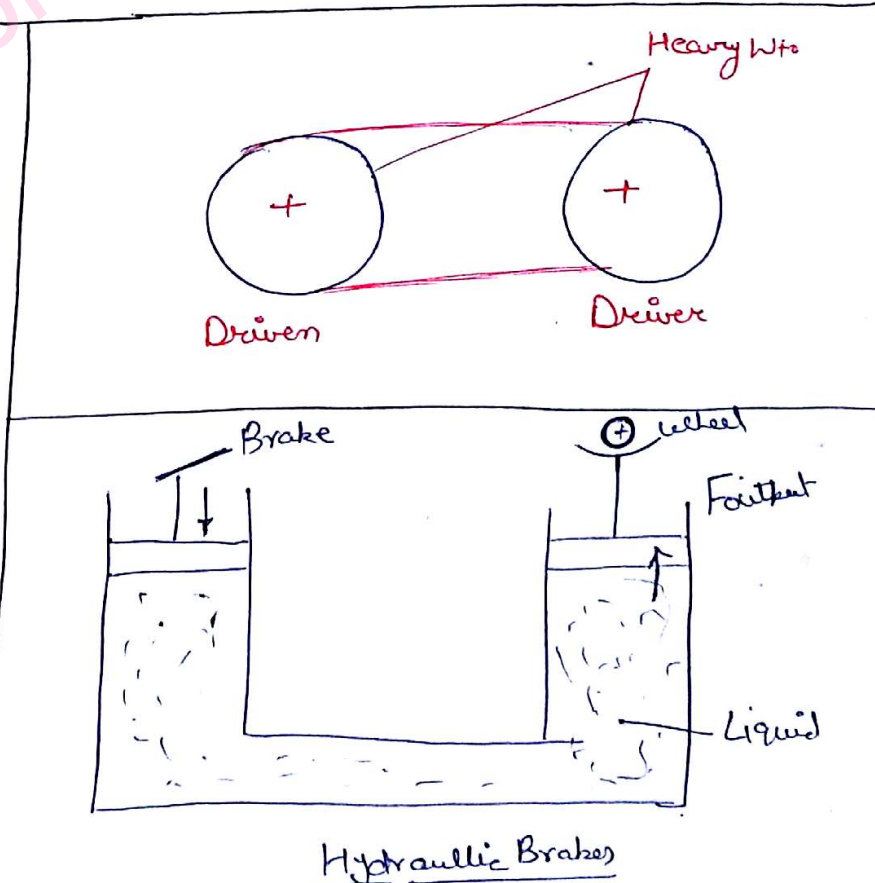
3) Fluid Link :-



When Power is Transmitted bc3 of fluid pressure.

for e.g.:-

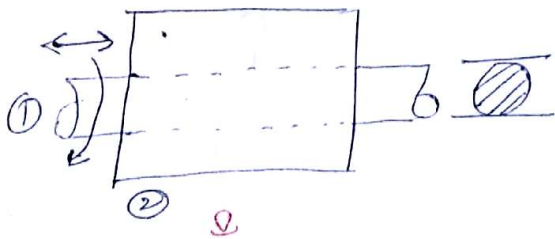
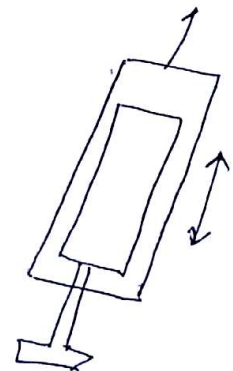
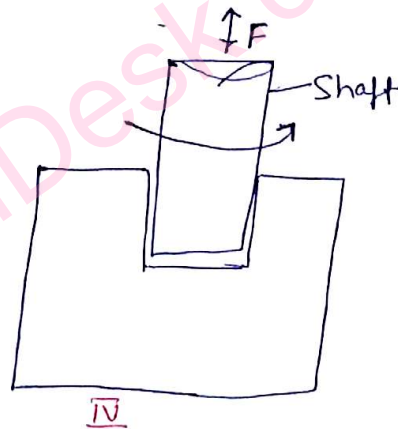
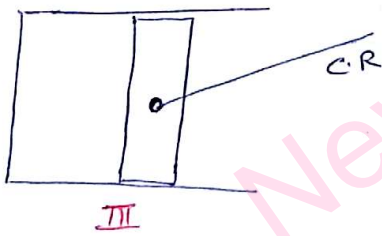
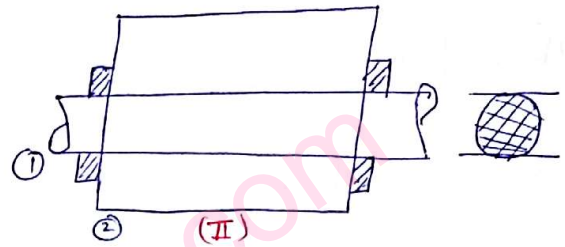
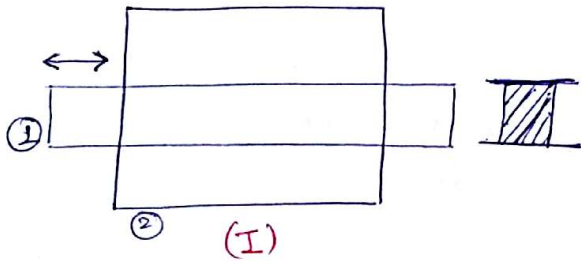
Hydraulic lift
Hydraulic RAM
Hydraulic crane etc



Different Types of Relative Motions :

Relative Motion \rightarrow For Relative motion system is having Two links.

- 1) Completely Constrained motion
 - 2) Successfully Constrained Motion
 - 3) In Completely Constrained Motion \rightarrow Unconstrained Motion
- } Constrained Motion $x \rightarrow y$
(Desired Motion)



- i, ii) Completely Constrained Motion
- iii, iv) Successfully Constrained
- v) In Completely Constrained

Kinematic Pairs:

" Any Connections b/w two links is always a joint or a pair, but this pair will also be a Kinematic pair, if the Relative motion b/w the links is a Constrained motion.

Classification of Kinematic Pairs:-

A) According to the type of Relative motion:-

i) Turning Pair (Revolute Pair) (Pin joint):

Kinematic has one independent

When Relative motion is pure turning.

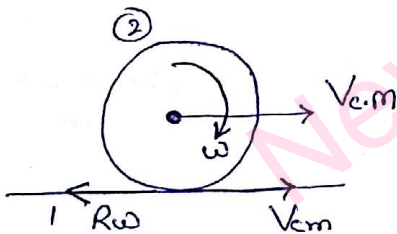
ii) Sliding Pair (Prismatic Pair):

When Relative motion is pure sliding.

iii) Rolling Pair:

When Relative motion is pure Rolling.

Rolling without slipping.



For Pure rolling
 $V_{cm} = R\omega$

iv) Screw Pair:-

When relative motion is over the threads.

for e.g. Nut & Bolt.

v) Spherical Pair (Ball in Socket joint): 3D Rotation

When Relative motion is (3-D) Rotation. Spherical Motion.



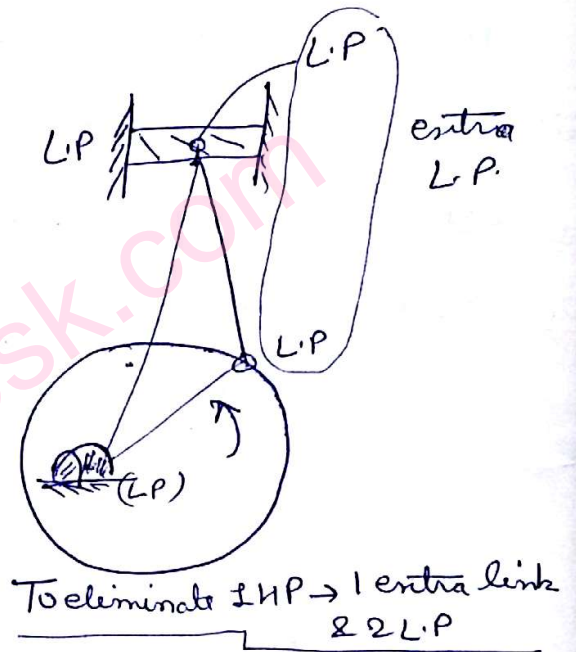
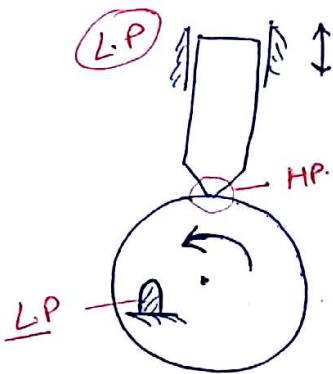
e.g. mirror in vehicles

B) According to Type of Contact :

- i) Lower Pair : → Surface Contact
- ii) Higher Pair → Point / line Contact
- iii) Wrapping Pair → When one link is wrapped over other link
Wrapping Pair is close to Higher Pair.

e.g → Belt Pulley, Shoes Shall, Rope Pulley.

$$\boxed{1HP \equiv 2LP}$$



c) According to the type of closure :

i) Self closed Pair (closed Pair) :-

Permanent Contact

e.g → Ball in Ball Bearing,



ii) Forced closed Pair (open Pair) :-

forceful Contact.

e.g: HP is Cam & follower

Door closer

Automatic clutch operating system



Centre of force

as C.F ↑ clutch operates & gears changes.

Operation depend on Springs & Cam

Different Types of joints :

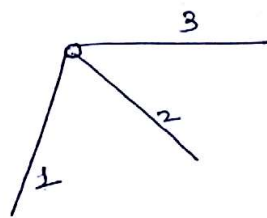
1) Binary joints :

When two links are connected.



2) Ternary joints :

When three links are connected.

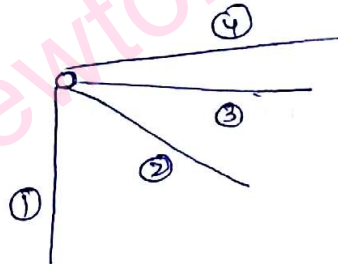


$(1, 2) \rightarrow B$
 $(2, 3) \rightarrow B$
 $(1, 3) \rightarrow B$
 ↓
 Dependent.

$$1T \equiv 2B$$

3) Quaternary joint :

When four links are connected.

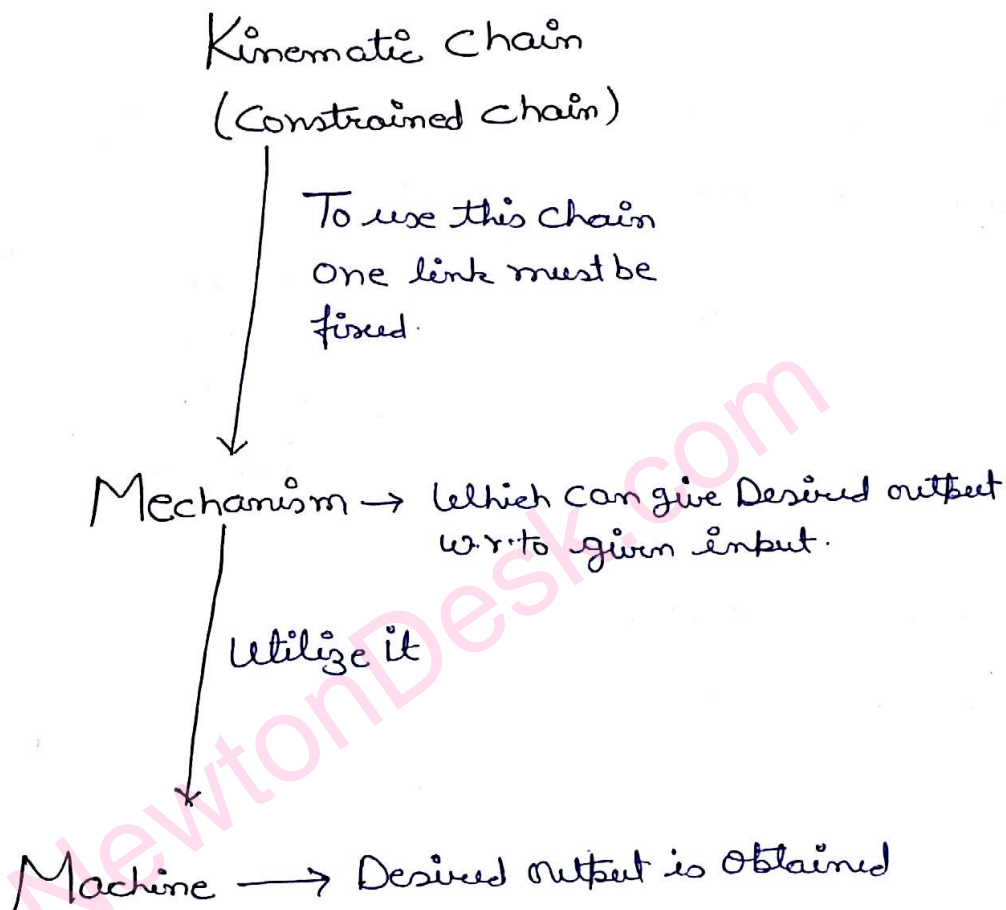


$(1, 2)$ $(2, 3)$ $(1, 4)$
 $(2, 3)$ $(2, 4)$
 $(3, 4)$

$$1Q \equiv 3B$$

→ Kinematic chain :

" If all the links are connected in such a way such that first link connected to last link in order to get the close chain and if all the relative motion in this close chain are constrained, then the chain is known as Kinematic chain.





Degrees of Freedom: (Mobility)

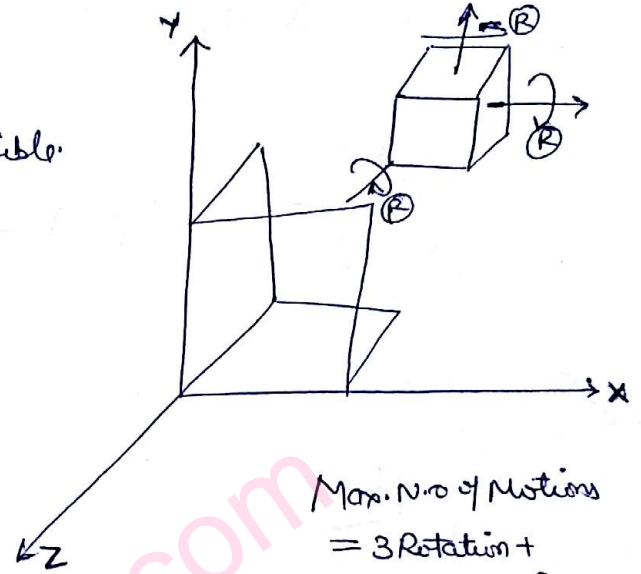
"The min. no. of independent variables requires to defines the position & motion of system is known as degrees of freedom of the system."

$$D.O.F = 6 - \text{Restrains}$$

↓
No. of those motion
which are not possible.

$$\frac{1}{\sigma \tau \xi} \frac{1}{\sigma \tau \xi} \frac{1}{\sigma \tau \xi}$$

Pair	Restrains	D.O.F
	$3T + 2R = 5$	$6 - 5 = 1$
	$1T = 1$	$6 - 1 = 5$



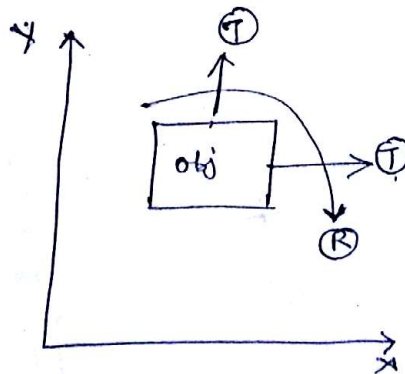
Max. N.o of Motions
= 3 Rotation +
3 Translation

Note

- 1) Lower Pair → 1 D.O.F
- 2) Higher Pair → 2 D.O.F
- ③ Spherical Pair → D.O.F = 3
bec it can rotate in 3 dirⁿ.

Aim:

To find out degree of freedom of (2-D) Planer Mechanism.



$$\text{Motion} = 2T + 1R = 3$$

No. of links = l

No. of Binary = j
Joints

No. of Higher = h
Joints

One link fixed

$$F = 3(l-1) - 2j - h \quad **$$

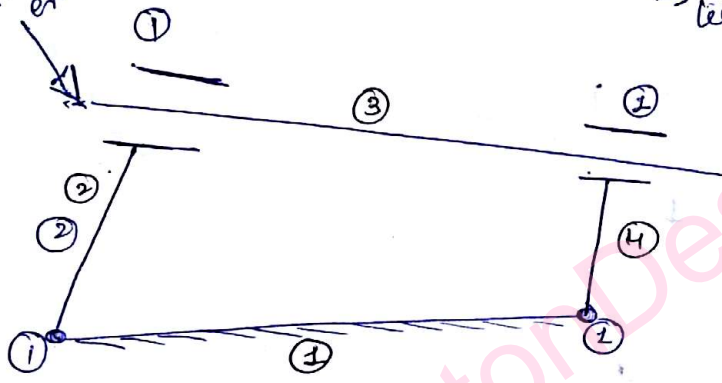
No. of Max. motion
in 2-D
Planar Mechanism

$$F = [3(l-1) - 2j - h] - F_r$$

Without any input
when this link
move then
external force
is applied

Where F_r = No. of those machines which
are not the Part of Mechanism.
Without input = source

1)

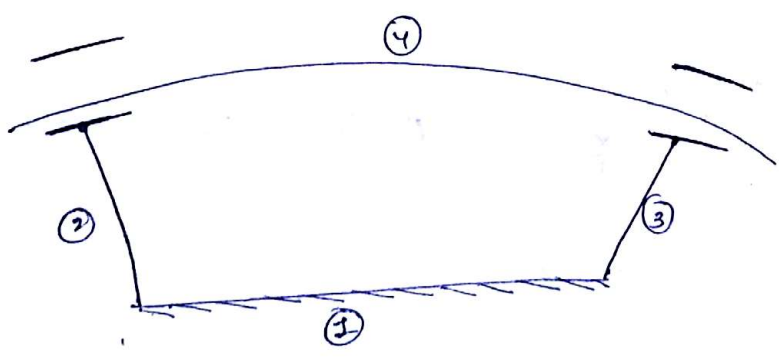


$l = 4$
 $j = 4$
 $h = 0$
 $F_r = 1$

$$F = [3 \times (4-1) - 2 \times 4 - 0] - 1$$

$F = 0$

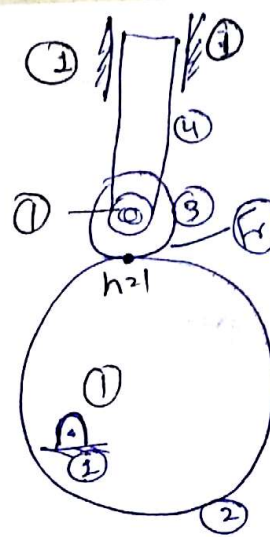
2)



$l = 4$
 $j = 4$
 $h = 0$
 $F_r = 0$
 $F = 1$

③ $F = [3(4-1) - 2 \times 3 - 1] - 1$

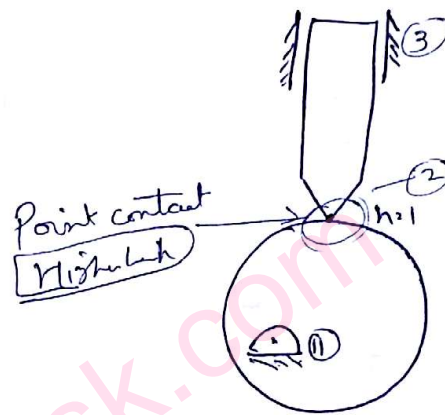
$F = 1$



$l = 4$
 $j = 3$
 $h = 1$
 $F_r = 1$

④ $F = (3(4-1) - 2 \times 3 - 1)$

$F = 1$

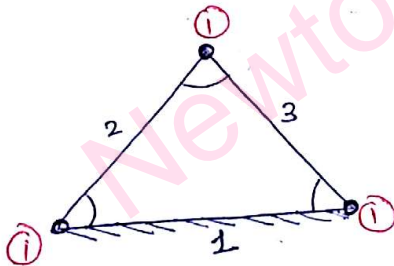


$l = 3$
 $j = 2$
 $h = 1$
 $F_r = 0$

Physical Significance of Degree of freedom:

1) If $F = 0 \rightarrow$ No Relative Motion (Frame/Structure)

e.g.:

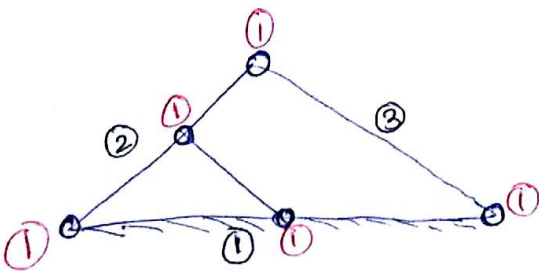


$l = 3$
 $j = 3$
 $h = 0$
 $F = 3(l-1) - 2j - h - F_r$
 $= 3(3-1) - 2 \times 3 - 0 - 0$
 $= 6 - 6$
 $F = 0$

$F = 0$

2) If $F < 0; \rightarrow -1, -2, -3,$

No Relative motion Super Structure
 (Intermediate Structure)
 (with great strength)

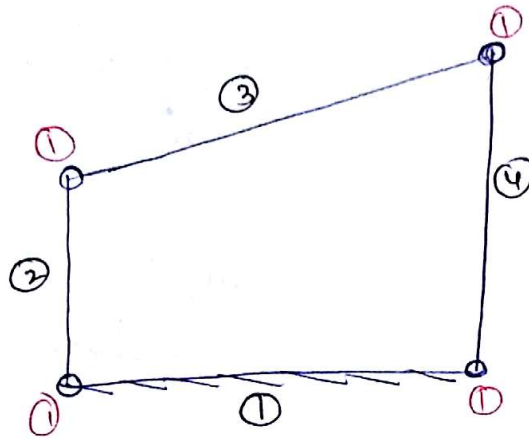


$l = 4$
 $j = 5$
 $h = 0$
 $F = -1$ ✗

3) If $F=1$

Kinematic chain

E.g.



$$L = 4$$

$$J = 4$$

$$h = 0$$

$$F = 1 \text{ * K.C.}$$

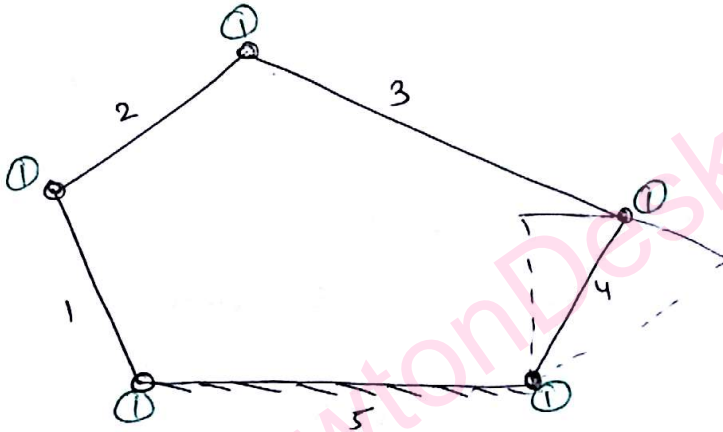
Kinematic chain

$$DOF = 1$$

~~***~~

4) If $F > 1$ 2, 3, 4, 5 — Unconstrained chain

E.g.



$$L = 5$$

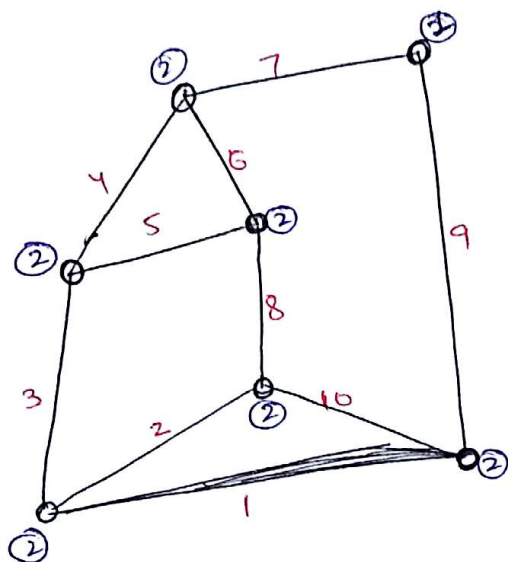
$$J = 5$$

$$h = 0$$

$$F = 2$$

" DOF is the no. of input Required to get the unconstrained output in Any chain.

ii)



$l = 10$

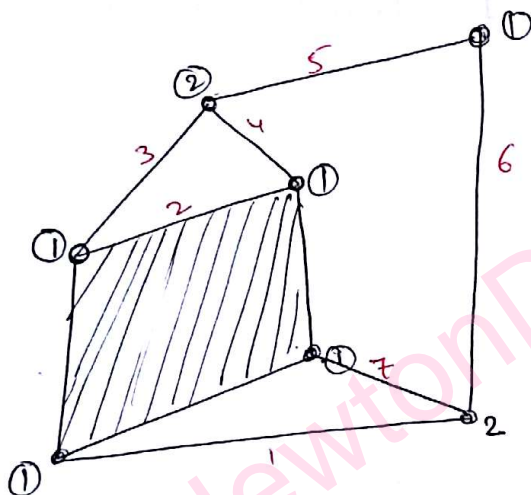
$j = 13$

$h = 0$

$F = 1$

Kinematic chain

iii)

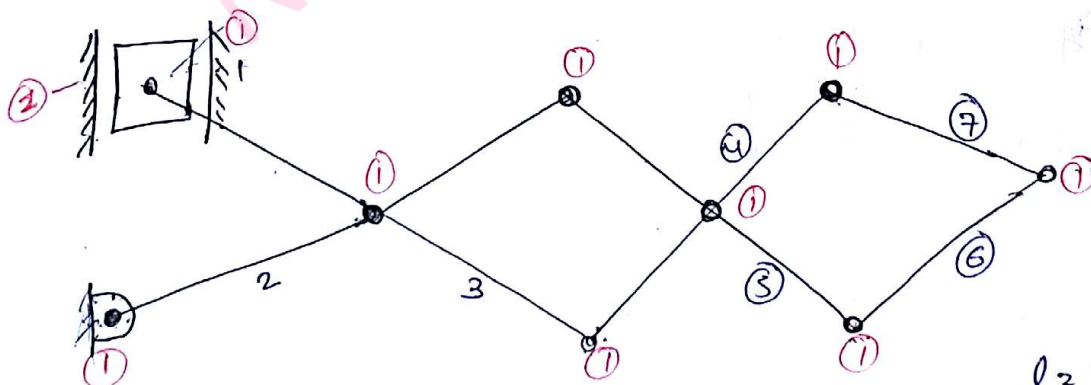


$l = 7$
 $j = 9$
 $h = 0$

$F = 0$

frame/structure

iv)



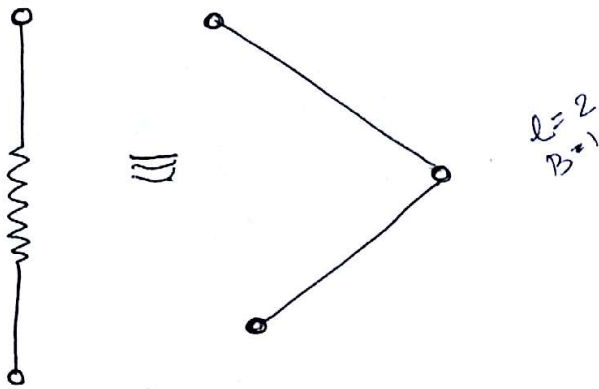
fixed $\rightarrow 1$
1 Sliders

Sliders in cylinders
1 joint

$l = 8$
 $\rightarrow j = 10$
 $h = 0$

$F = 1$ (KOC)

Spring as a link Vermb



Gruebler's Equation:

Gruebler's equation is valid only for those mechanism in which

$$F=1$$

$$h=0$$

Applying Kutzbach's eqn:

$$F = 3(l-1) - 2j - h$$

$$1 = 3l - 3 - 2j = 0$$

$$3l - 2j - 4 = 0$$

$(3l)$ always \rightarrow even

(l) always \rightarrow even

$$(l)_{\min} \rightarrow 4$$

first Mechanism in Lower Pairs

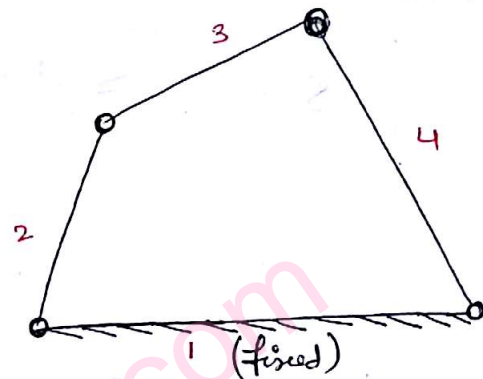
Simple Mechanism :-

- i) Four bar Mechanism
- ii) Single Slider Crank Mechanism
- iii) Double Slider Crank Mechanism

} Four link
(Lower Pair)

i) Four bar Mechanism :- (4 Links, 4 Turning Pairs)

Best Position \rightarrow fixed
because it governs both
input and output



input/output
 \Downarrow

- Complete rotation \rightarrow Crank (Complete 360°)
- Partial Reaction \rightarrow Rocker/lever (if less than 360°)
oscillation

Voip (Note)
If NO. of Links = l
then NO. of inversions $\leq l$

Inversions :-

Mechanism which are obtained by fixing one by one different links

- i) Double Crank Mechanism
- ii) Crank \leftrightarrow Rocker Mechanism
- iii) Double Rocker Mechanism

Gyashof's Law:

"^{Aim} For the Continuous Relative motion b/w the number of Planes, in a mechanism, the Summision of ~~any~~ length of Shortest & the longest link should not be greater than the Summision of the length of other two links!"

** For Continuous Relative motion

$$(S + l) \leq (P + q)$$

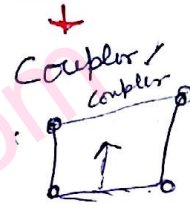
S - Shortest link
l - longest link

Where,

Best Position \rightarrow fixed $\begin{cases} \text{input} \\ \text{output} \end{cases}$

Best link for Rotation \rightarrow S

Closest Position



Case 1 if $(S+l) < (P+q)$

Law is satisfied

1) S \rightarrow fixed $\begin{cases} \text{input} \\ \text{output} \end{cases} \rightarrow$ Double cranks.

2) S \rightarrow Adjacent Link fixed \rightarrow Crank Rocker

3) S \rightarrow Coupler \rightarrow Double Rocker.

Case 2 if $(S+l) = (P+q)$

Law satisfied

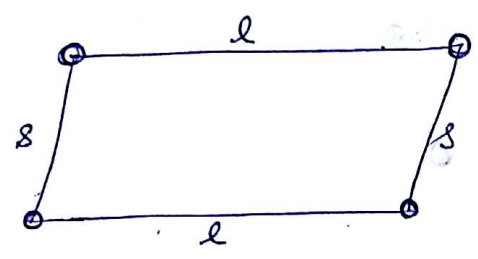
Condition: Not having Pair of equal links



Case 3 if $(s+l) = P+q$ but, Having equal links

2, 2, 5 5
s s L L

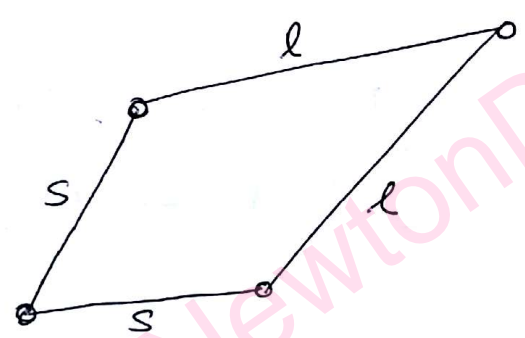
1) Parallelogram linkage:



s-fixed
↓
Double Crank

l-fixed → Double Crank.
bcz short length can rotate easily.

2) Deltoid linkage:



Short-fixed
↓
s-fixed
↓
Double Crank

l-fixed → Crank-Rocker.

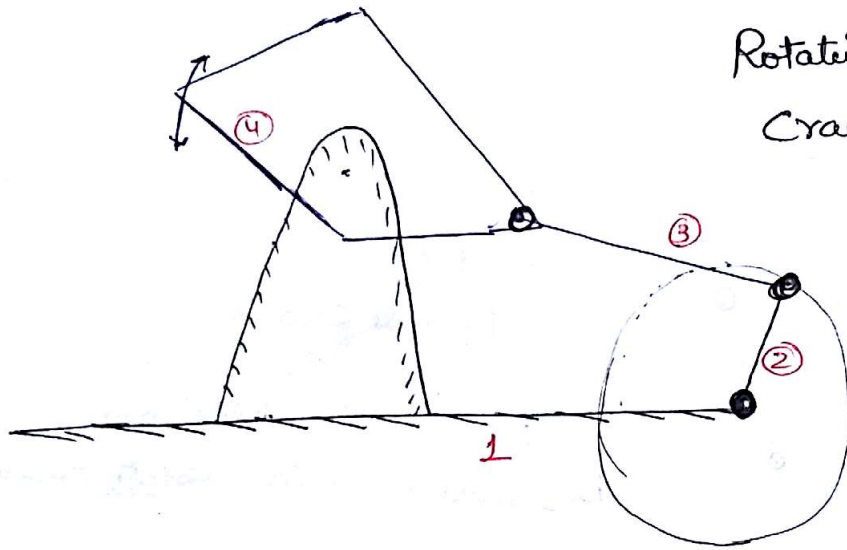
if $(s+l) > (P+q)$

Law not satisfied

Double-Rocker will be obtained

Some Practical Applications of Four bar Mechanism

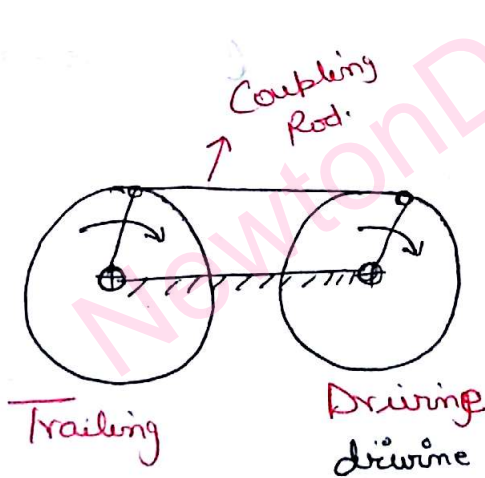
1) Beam Engine Mechanism: By James Watt



Rotation \longleftrightarrow oscillation

Crank \longleftrightarrow Rocker.

2) Coupling Rod of Locomotive:



[Parallelogram linkage
Double crank]

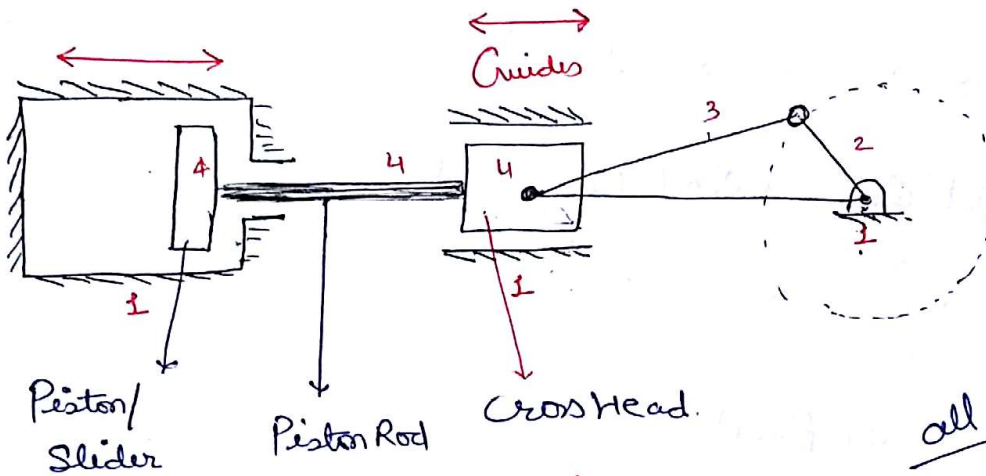
Single Slider - Crank Mechanism: 4 link = 3TP + 1SP

(ii) Inversions

TP → Turning Pair
SP → Sliding Pair

(i) I, inversion (Basic) (Cylinder fixed)

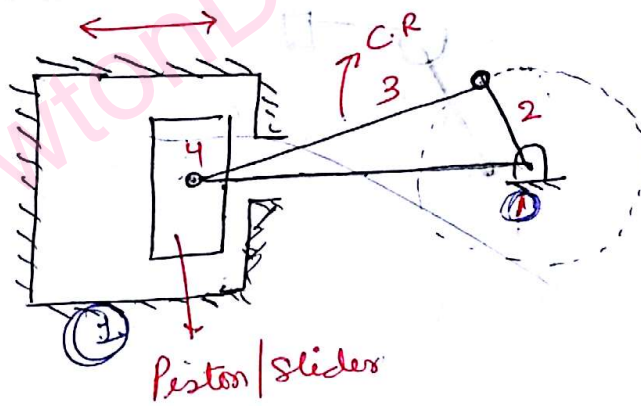
external Combustion engine



all fixed taken as one link
joint may be same

Same.

Internal Combustion engine



Summary

Rotation ↔ Reciprocating
(Crank) ↔ (Piston)

(0) ← i → Reciprocating engine
(i) → 0 → Reciprocating Compression

ii) II Inversion
(Crank fixed)

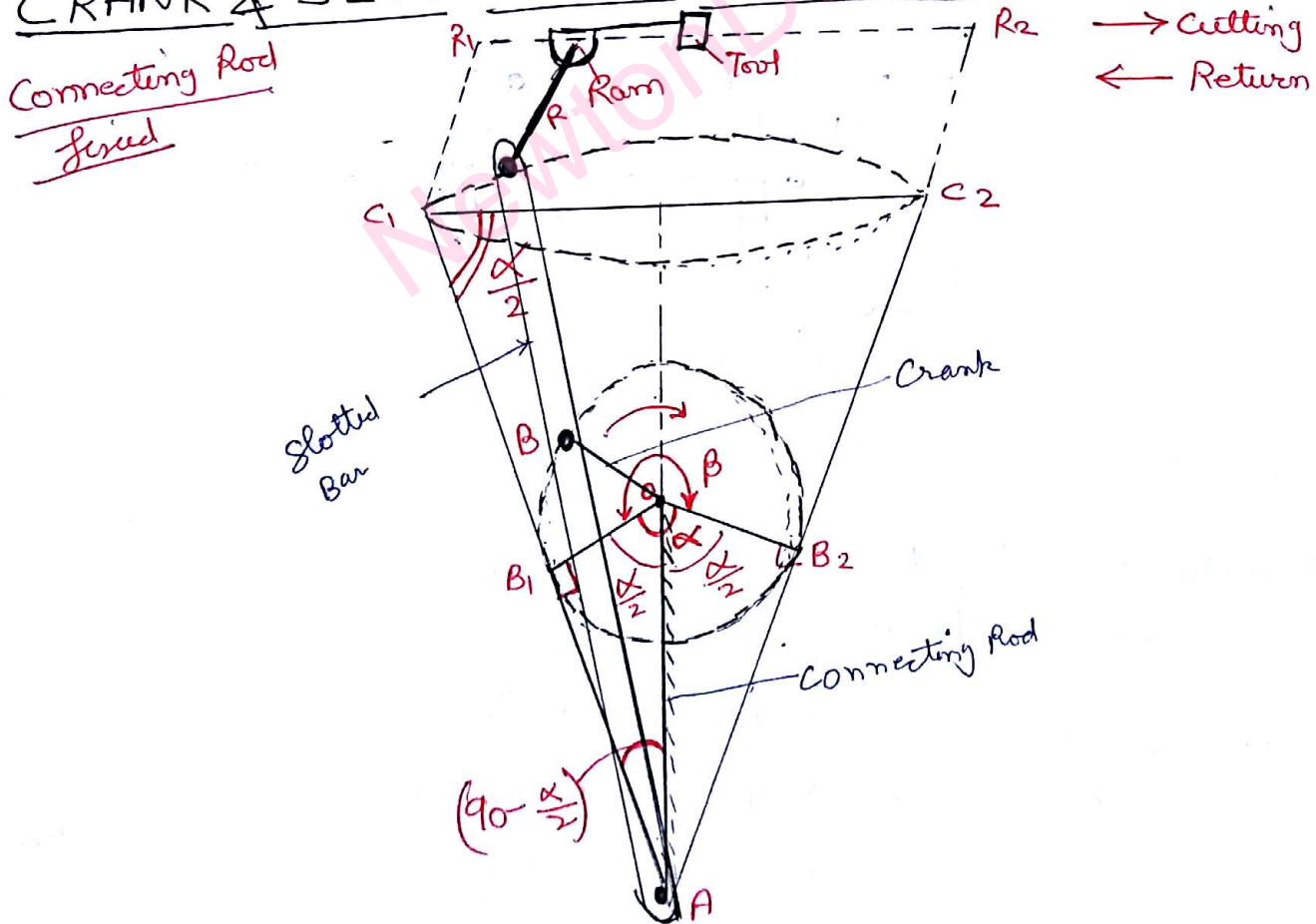
- i) Whitworth (Quick Return Motion Mechanism) & RMM
- ii) Rotary I.C engine Mechanism (GNOME Engine)

iii) III Inversion
(~~Crank fixed~~) (Connecting Rod fixed)

- i) Crank & Slotted lever (QRMM) Best
- ii) Oscillating cylinder engine Mechanism.

iv) IV Inversion (Slider fixed)
Hand pump (Pendulum Pump) (Bull Engine)

CRANK & SLOTTED lever (QRMM) - (C & R fixed)



$\beta \rightarrow$ Cutting Stroke angle

$\alpha \rightarrow$ Return Stroke angle

$$\alpha + \beta = 360^\circ$$

$$\boxed{\alpha < \beta} \quad \text{DRMM}$$

$$\frac{(\text{time}) \text{ Cutting}}{(\text{time}) \text{ Return}} = \frac{\beta}{\alpha} = \text{DRMM}$$

अगर $\frac{\beta}{\alpha}$ less than 1 given है
तो Ratio $\frac{\alpha}{\beta}$ को दें

$$\boxed{\frac{\beta}{\alpha} > 1} \text{ not}$$

Note \rightarrow if in Question it is given less than one
then it is Ratio of $\frac{\alpha}{\beta}$ is given.

Stroke

$$= R_1 R_2$$

$$= 4 C_2$$

$$= 2(C_1 M)$$

$$= 2(AC_1) \cdot \cos \frac{\alpha}{2}$$

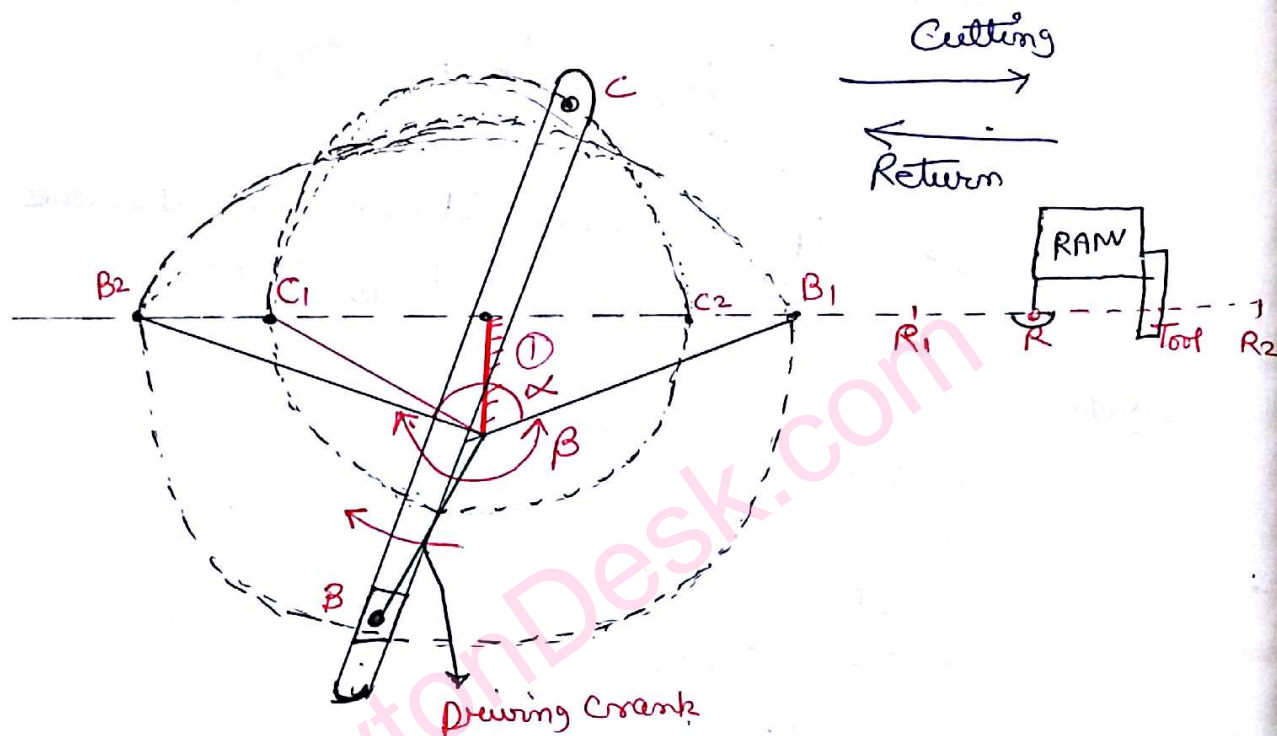
$$= 2(AC_1) \left(\frac{OB_1}{OA} \right)$$

$$= \frac{2(AC)(OB)}{(OA)}$$

$$= \frac{2(\text{Length of slotted bar}) \times (\text{Length of Cranks})}{\text{length of Connecting Rod.}}$$

Wielhworth Quick Return Motion Mechanism (Crank fixed)

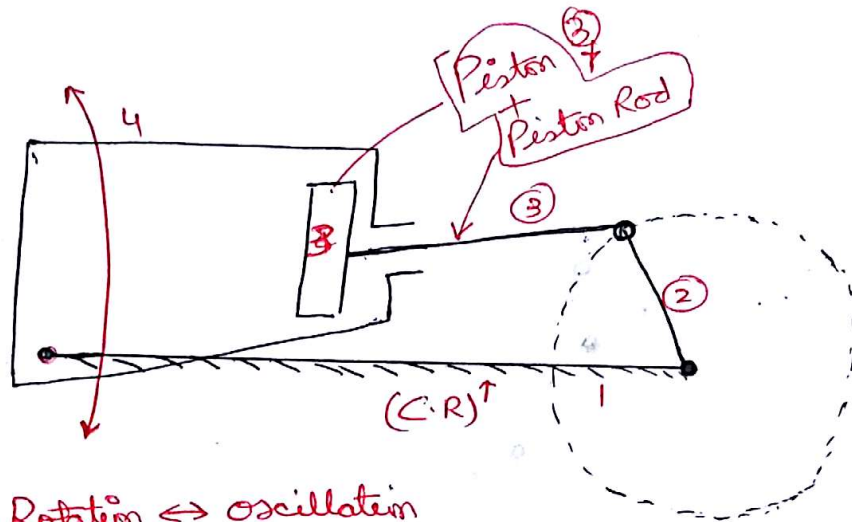
Rotation \rightarrow Rotation
 Double Crank.



Stroke

- = $R_1 R_2$
- = $C_1 C_2$
- = $2\theta_c$

Oscillating Cylinder Engine Mechanism: NewtonDesk.com (Connecting Rod fixed)



Rotation \leftrightarrow oscillation
Crank \leftrightarrow Rocker

Rotary Internal Combustion Engine: (GINONE Engine) (Crank fixed)

When combustion takes place inside the cylinder,

Input force comes on piston

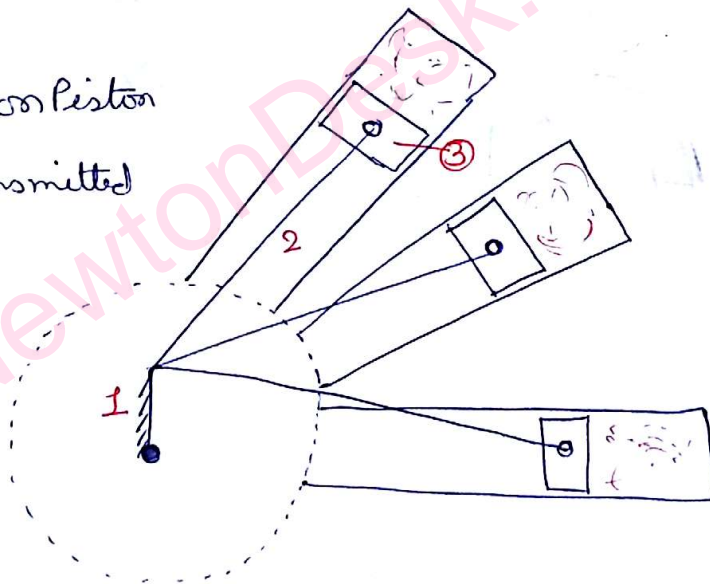
then this force transmitted to connecting rod.

then, connecting rod ~~gain this force.~~

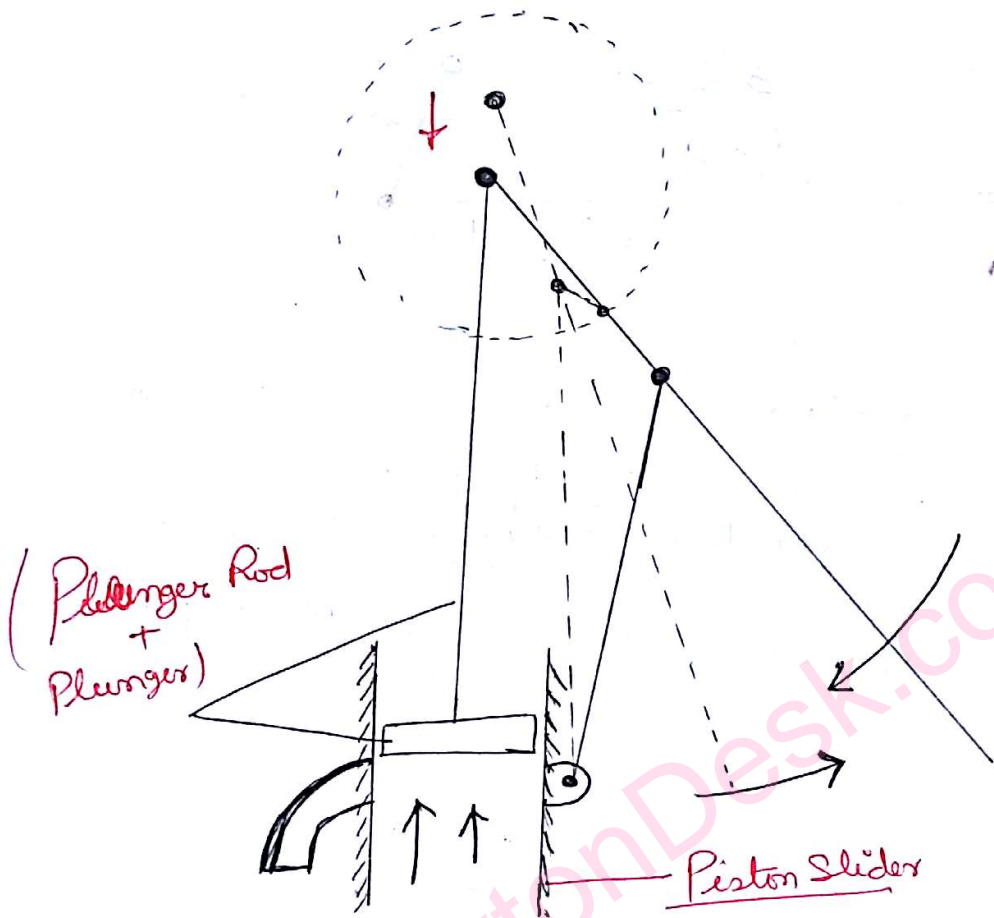
and piston both rotates



Cy. Block rotates (output)

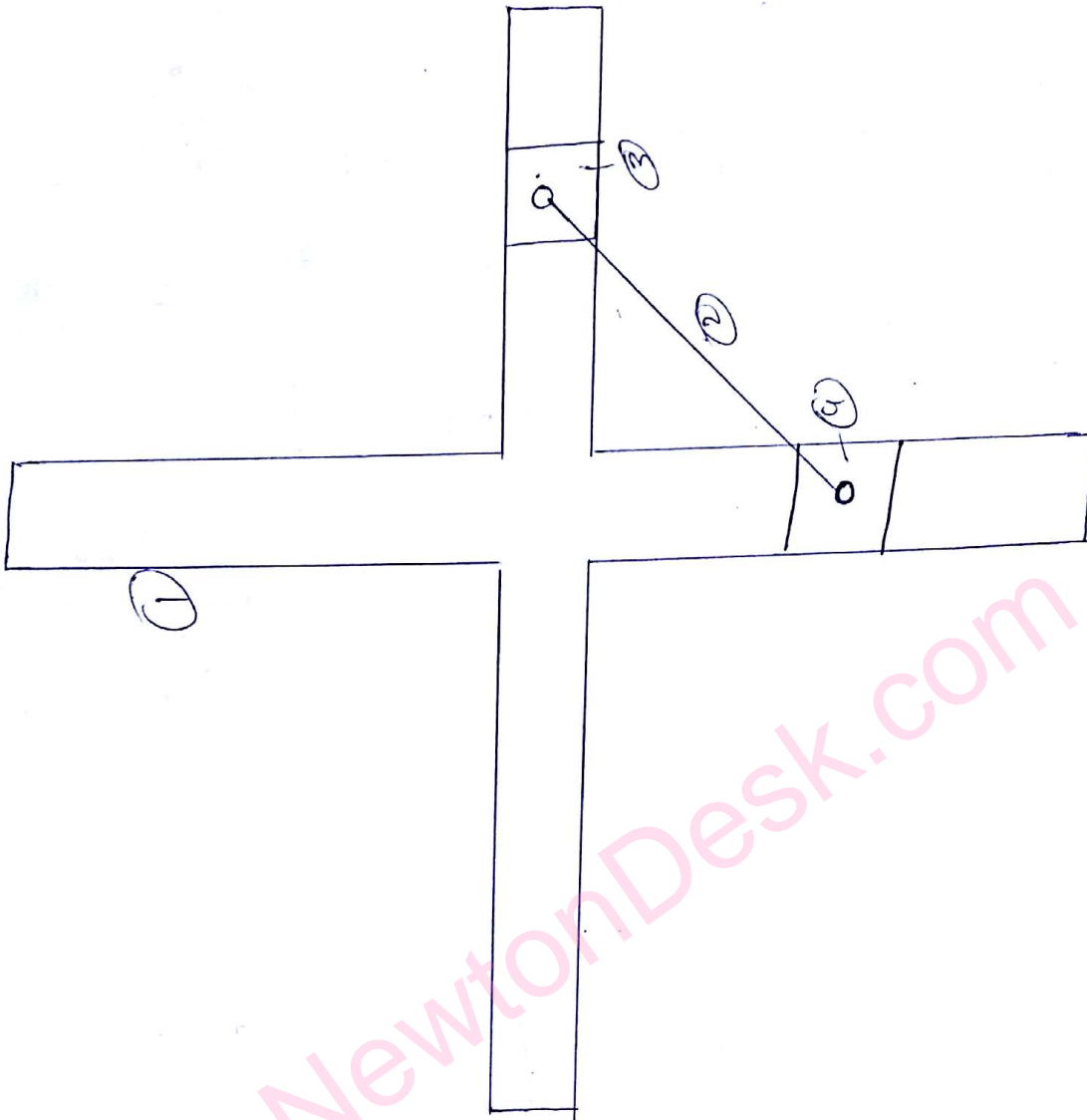


Hand Pump (Slider Pump) :-



③ Double Slider-Crank Chain

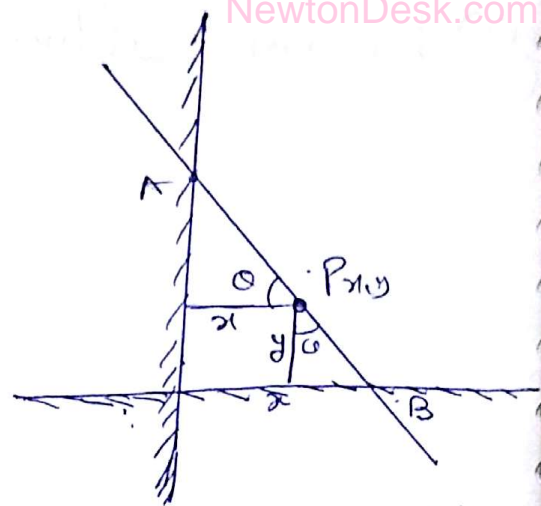
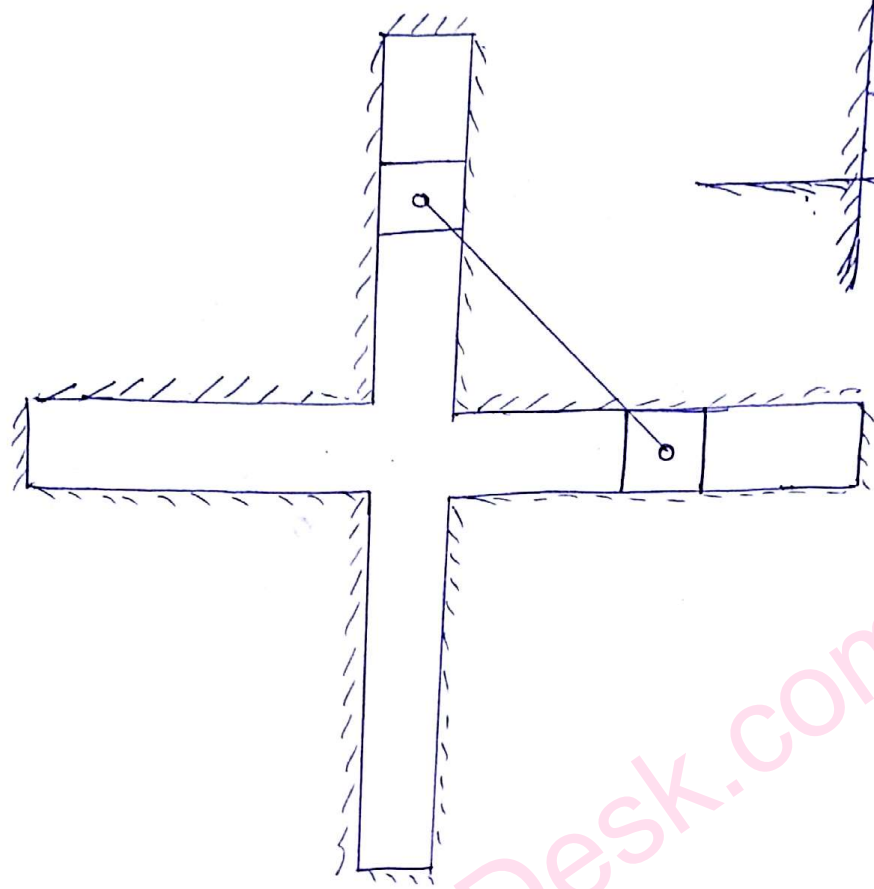
4 link \rightarrow 2TP + 2SP



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1) Slotted Plate fixed:

Elliptical Trammels:

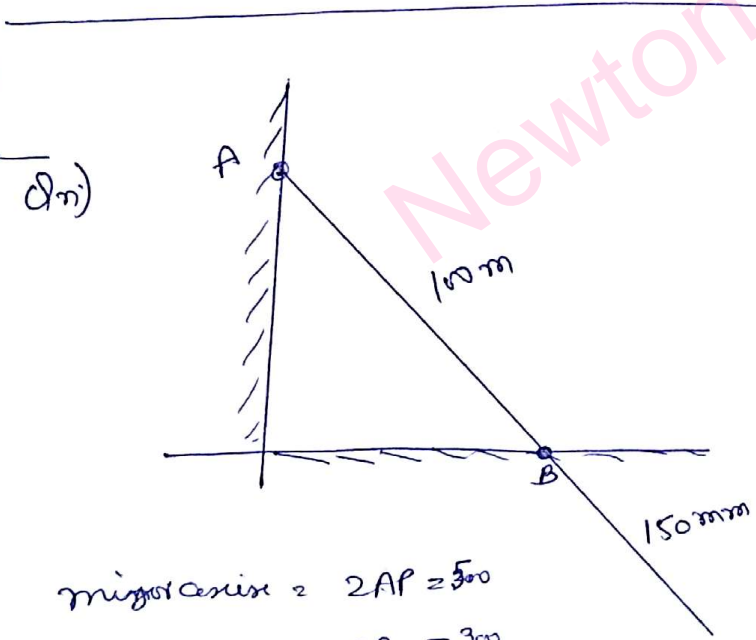


$$\cos \theta = \frac{x}{AP}$$

$$\sin \theta = \frac{y}{BP}$$

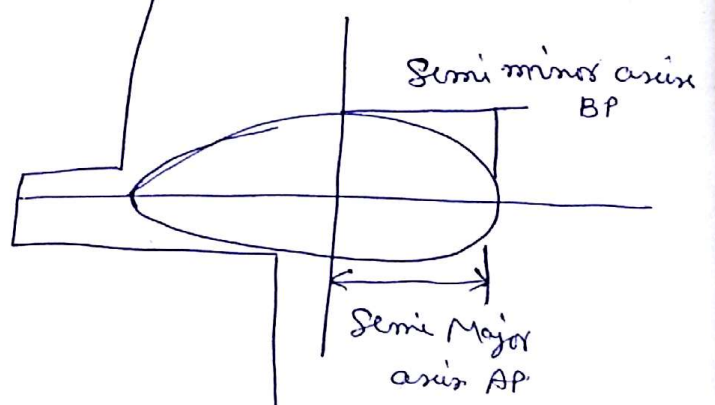
$$\frac{x^2}{AP^2} + \frac{y^2}{BP^2} = 1$$

ellipse



major axis = $2AP = 300$

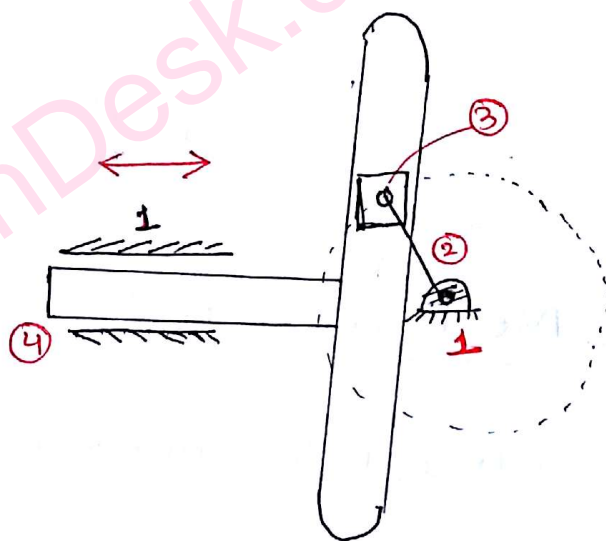
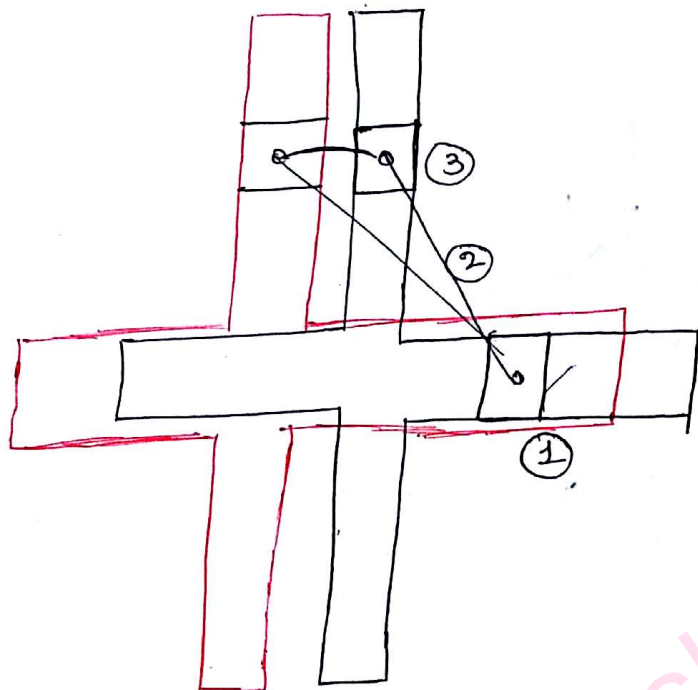
minor axis = $2BB = 300$



2) If Any of the Slides is fixed :-

Scotch-Yoke Mechanism

Rotation \rightarrow Reciprocating

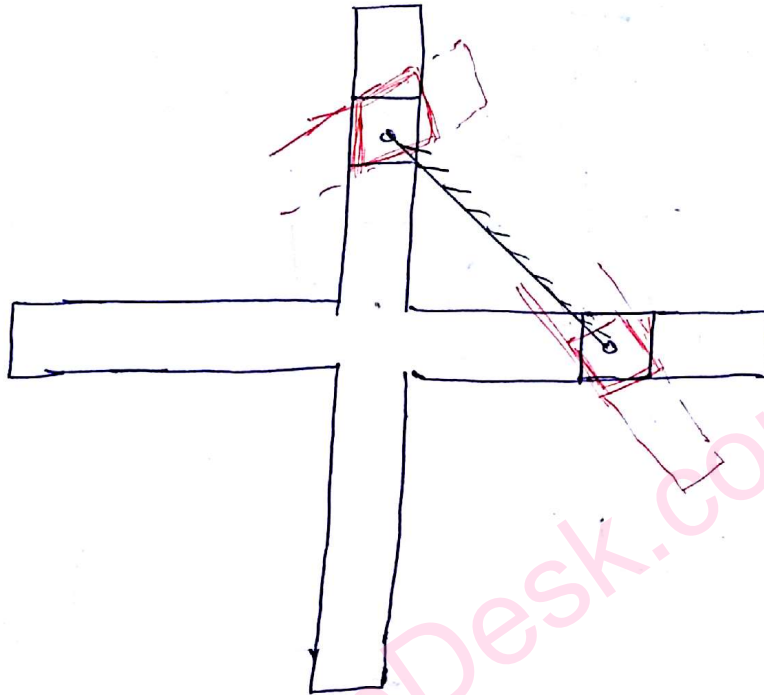


3) If Link Connecting slides is fixed:

Oldham's Coupling:



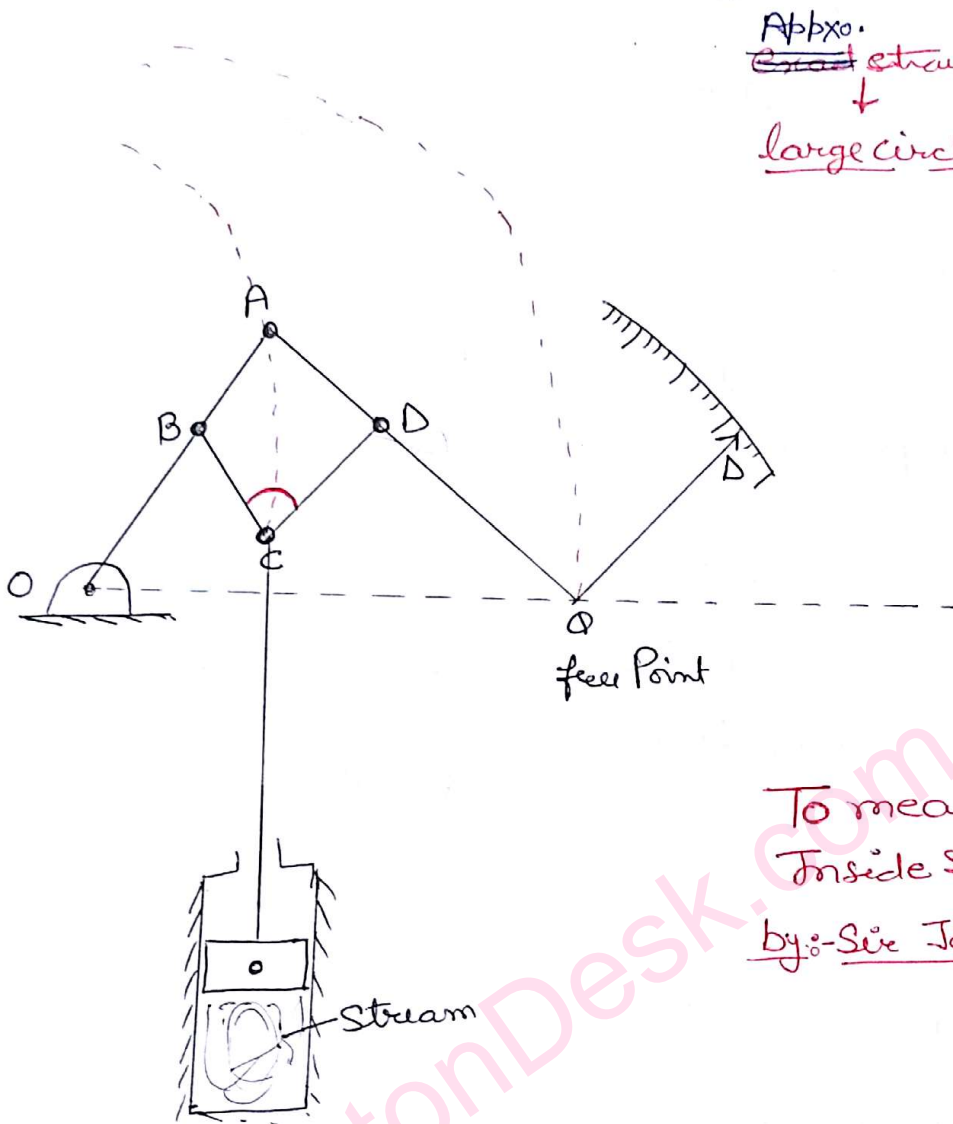
used to connect the shafts having lateral misalignment.



Complex Mechanism with Lower Pair:

- Exact straight line Motion Mechanism.
- Approximate straight line Motion Mechanism.

Watt's Indicator Mechanism:-



Appxo.
~~circle~~ straight line
 ↓
large circle with ↑ radius

To measure Pressure
 Inside Steam Chamber
 by:- Sir James Watt

Observation:

- Point C and Point Q, both moves in Approximate straight line motion. [Approximate st. line motion Mechanism]
- There is no relative motion b/w link BC & CD hence BCD → one link.
- Link BCD → levers → Double crank Mechanism
- Link AQ →

Steering Gear Mechanism:



Changing direction of motion.

i) Davis Steering gear Mechanism:

- having 2 Turning pair + 2 Sliding pair (Life is less)
- Exact at all positions.

ii) Accerman steering gear Mechanism:

- having only turning pairs (Life is very high)
- Exact at all three positions.
(mid, extreme left, extreme right)

iii) Rapson's Slid:

- used in ships

Intermittent Motion Mechanism:



Provide the periodic motions with breaks at output w.r. to given continuous input.

i) Geneva Mechanism:

- used in indexing in milling etc

ii) Ratchet Mechanism:

- used in clocks

Mechanical Advantage of A Mechanism

$$M.A = \frac{F_{\text{output}}}{F_{\text{input}}} \rightarrow \boxed{M.A = \frac{V_{\text{input}}}{V_{\text{output}}} \times \eta_{\text{mechanism}}}$$

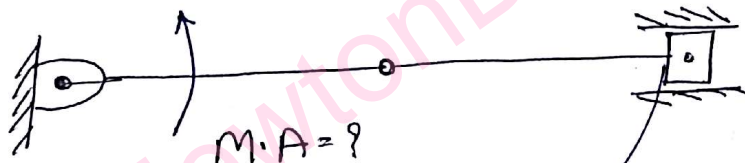
or

$$M.A = \frac{T_{\text{output}}}{T_{\text{input}}} \rightarrow \boxed{M.A = \frac{W_{\text{input}}}{W_{\text{output}}} \times \eta_{\text{mechanism}}}$$

Efficiency

$$\eta_{\text{mechanism}} = \frac{P_{\text{output}}}{P_{\text{input}}} = \frac{F_{\text{output}} \times V_{\text{output}}}{F_{\text{input}} \times V_{\text{input}}} = \frac{T_{\text{output}} \times \omega_{\text{output}}}{T_{\text{input}} \times \omega_{\text{input}}}$$

Qm) find Mechanical Advantage of leg?



Given that,

$$\omega_{\text{input}} = \omega$$

$$\omega_{\text{output}} = 0$$

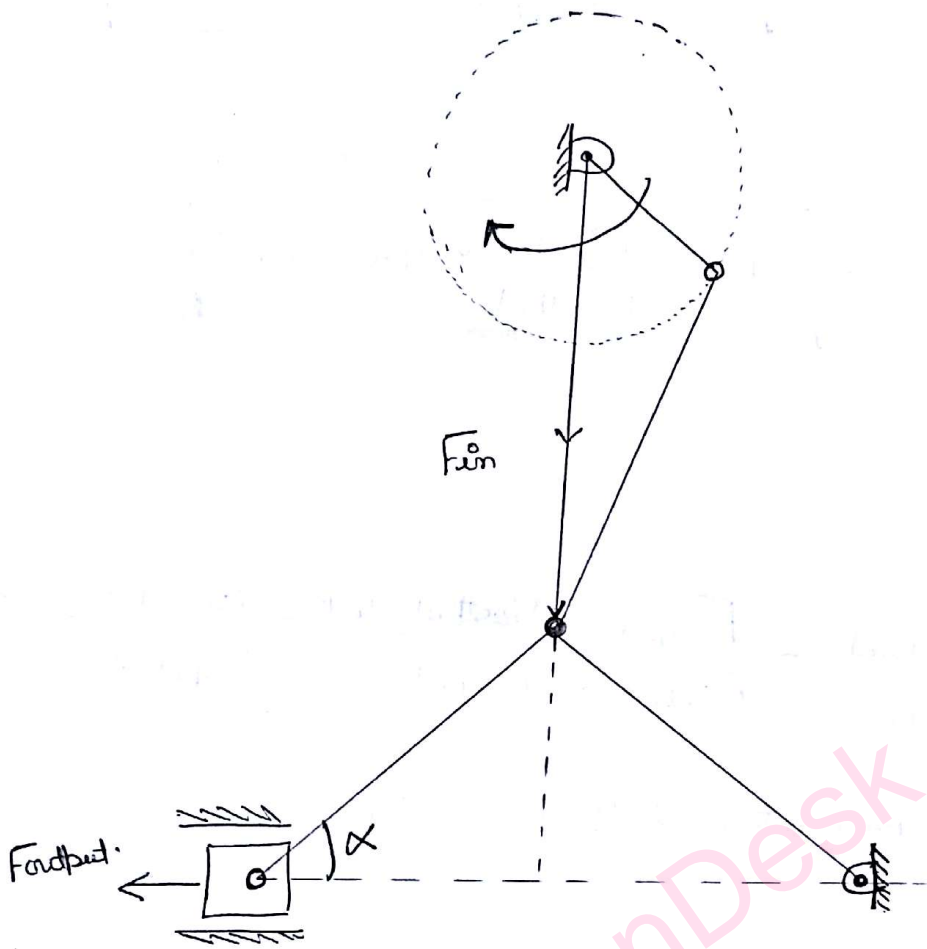
Pure sliding

$$\boxed{M.A = \frac{\omega}{0} = \infty}$$

BCz, Rotation \rightarrow Resiprocating

Rotation of infinite radius circle
Means Resiprocation.

Toggle Mechanism :-



$$\tan \alpha = \frac{F_{input}}{F_{output}}$$

$$F_{output} = \frac{F_{input}}{\tan \alpha}$$

As $\alpha \rightarrow 0$

$$\tan \alpha = 0$$

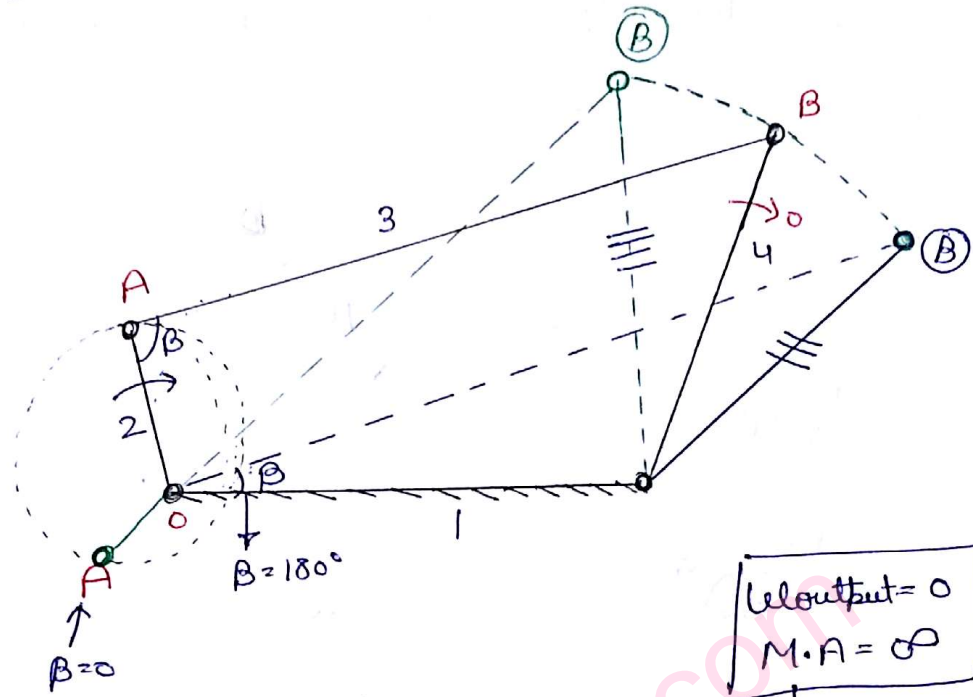
$$F_{output} \rightarrow \infty$$

$$M.A = \alpha$$

$$F_{output} \gg \gg \gg F_{input} *$$

Toggle's Position :

It is the extreme positions of output link, ~~As~~ Rocket in four Bar Mechanism.



$vel_{output} = 0$
 $M.A = \infty$

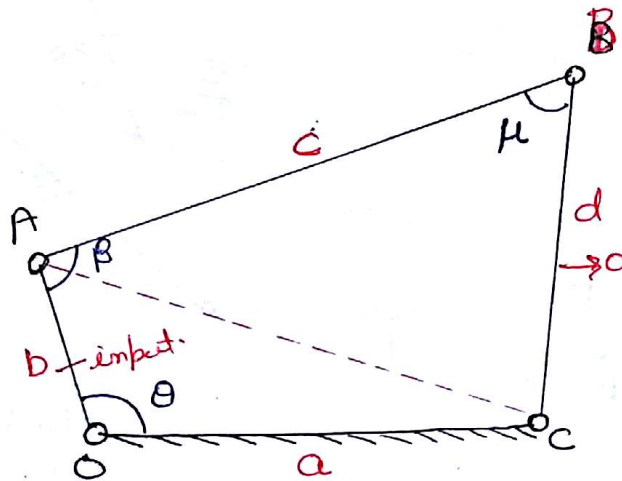
Toggle Position

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Transmission Angle in Four bar Mechanism : (H)

Representation of Transmission angle $\rightarrow H$

\rightarrow The Angle b/w the output link & Coupler link in four bar Mechanism is known as Transmission Angle.



$$AC^2 = b^2 + a^2 - 2ba \cos \theta = c^2 + d^2 - 2cd \cos H$$

Differentiating both side

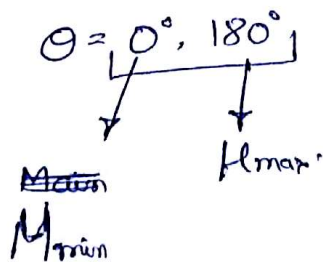
$$(-2ba) (-\sin \theta \cdot d\theta) = (-2cd) (-\sin H \cdot dH)$$

$$\frac{dH}{d\theta} = \frac{ba}{cd} = \frac{\sin \theta}{\sin H}$$

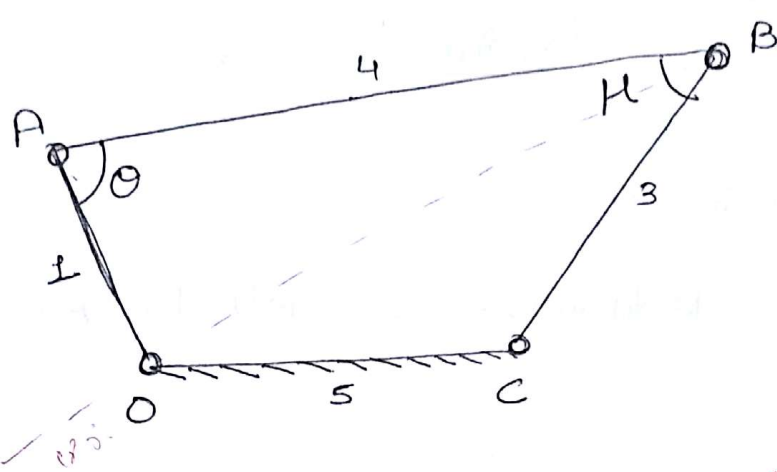
for H, to be max. or min.

$$\frac{dH}{d\theta} = 0$$

$$\Rightarrow \sin \theta = 0$$



Problem:



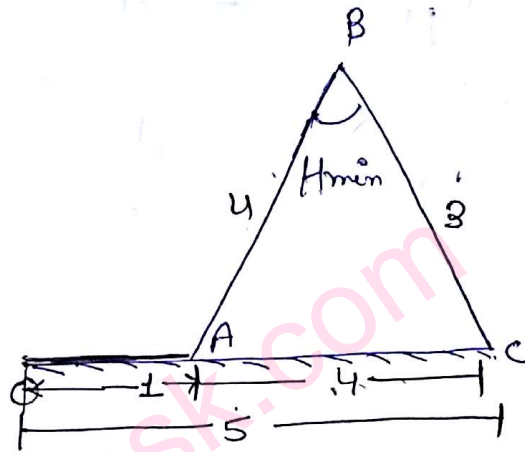
find, $H_{min} = ?$
 $H_{max} = ?$

first check whether triangle law is valid or not
 i. e. sum of lowest & Largest must be equal or less

for H_{min} ($\theta = 0^\circ$)

$$(4)^2 = (4)^2 + (3)^2 - 2(4)(3) \cos H_{min}$$

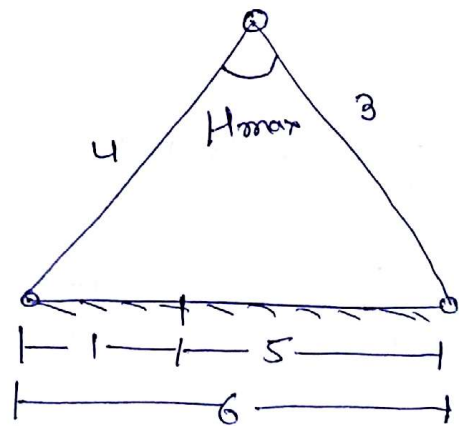
$$H_{min} =$$



for H_{max} ($\theta = 180^\circ$)

$$(6)^2 = (4)^2 + (3)^2 - 2(4)(3) \cdot \cos H_{max}$$

$$H_{max} =$$

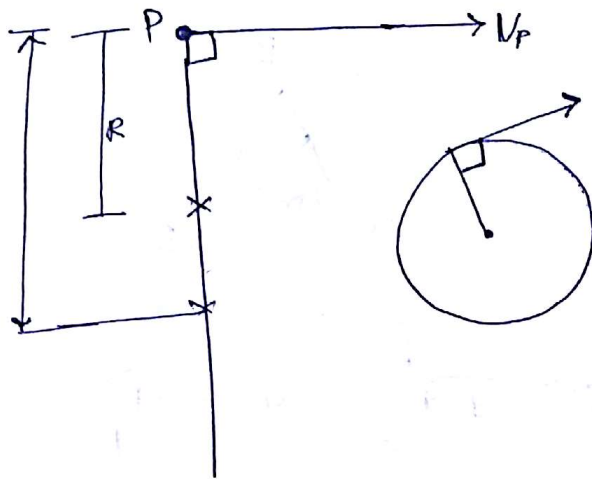


Velocity Analysis :-

Instantaneous Centre Method Application

(by: Sir, ARNHOLD)

Instantaneous Centre of Rotation :-



$$\frac{V}{R} = \omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \frac{V_D}{DI} = \frac{V_E}{EI} = \dots$$

for e.g.

$I_{23} \rightarrow$ Instantaneous Centre for the relative motion b/w two links i.e b/w link 2 & 3

I_{23} or I_{32} both are same. 2,3 represent link only

Motion	Centre	Axode
General Motion	Curve	Curved Surface
Pure Translation	Straight line	Plane Surface
Pure Rotation	Point	Straight line

In Reality

$$\left. \begin{array}{l} AA_1 \rightarrow 0 \\ BB_1 \rightarrow 0 \end{array} \right\} \text{Differential}$$

This link AB at this instant is in general motion

In general, when the link moves then its instantaneous centre of relative motion changes its position.

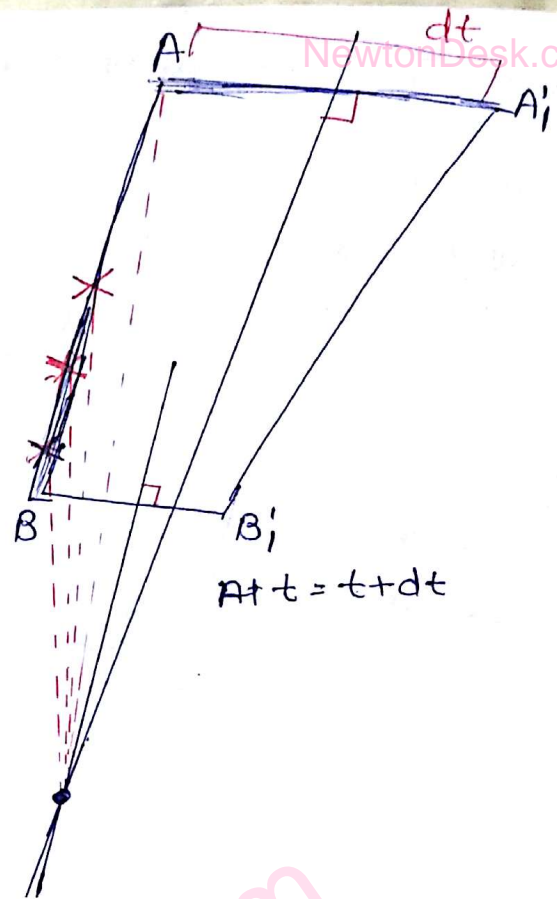
Locus of I-Centre for the relative motion b/w the Centrode link.

Locus of I-axis of Relation for the relative motion \Rightarrow Axode b/w the links

Motion

" In General, the motion of link in mechanism is neither pure translation nor pure rotation. It is a combination of translation and rotation, which we normally say, that the link is in general motion but, any link at any instant, can be assumed to be in pure rotation with respect to point in the space

Known as Instantaneous Centre of Rotation.
This Centre is also known as the Virtual Centre. "



No. of Instantaneous Centre in Mechanism:

If No. of link = l

$$\begin{aligned} \text{No. of Instantaneous Centre} &= lC_2 \\ &= \frac{l(l-1)}{2} \end{aligned}$$

for e.g.

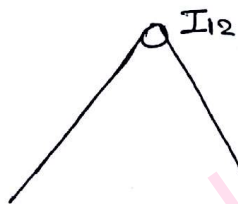
$$l = 5$$

$$I.C = 10$$

I_{12}	I_{13}	I_{14}	I_{15}
	I_{23}	I_{24}	I_{25}
		I_{34}	I_{35}
			I_{45}

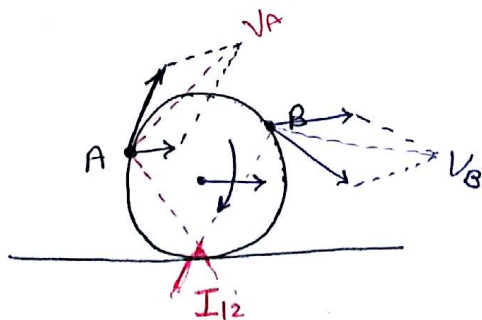
Basic Instantaneous Centre in a Mechanism:

1) Turning Pair:



$I.C$ at Centre / Point of Pin.

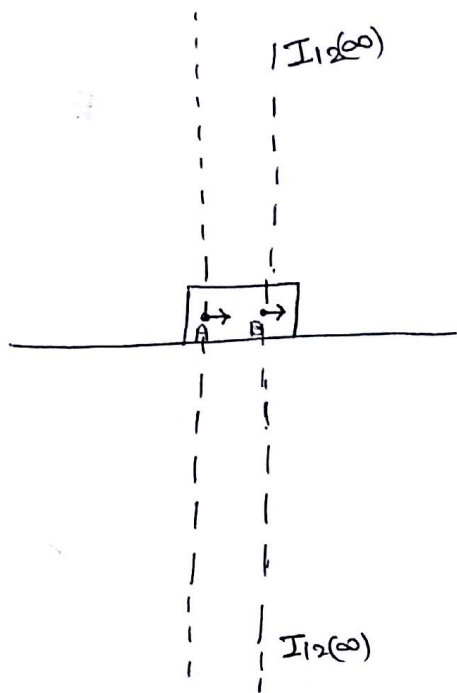
2) Rolling Pair:



③ Sliding Pair

i) on Plane Surface:

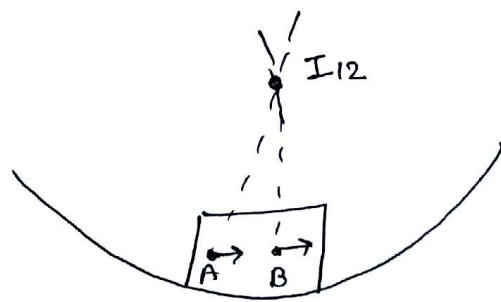
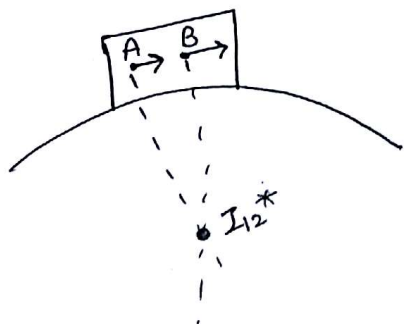
I_C will go at ∞ , but in the opposite dirⁿ, \perp to the sliding surface.



ii) Curved Surface:

Convex

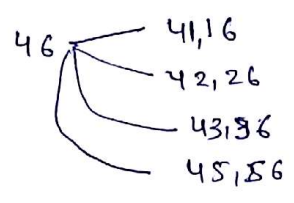
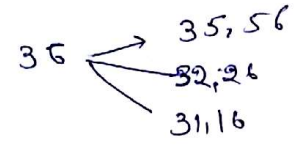
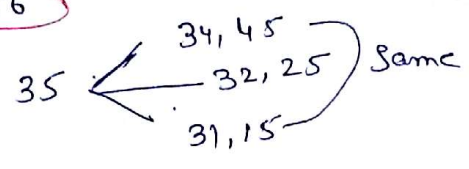
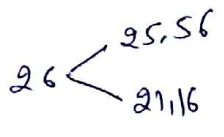
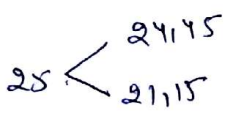
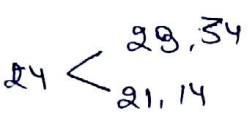
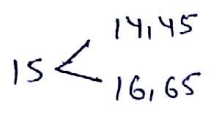
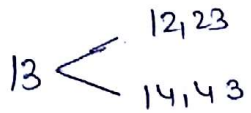
Concave



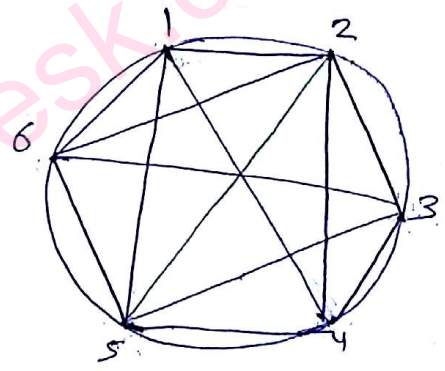
$l = 6$

$I_c = 15$

- I_{12} I_{13} I_{14} I_{15} I_{16}
- I_{23} I_{24} I_{25} I_{26}
- I_{34} I_{35} I_{36}
- I_{45} I_{46}
- I_{56}



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Crewin

$$N_{OA} = 120 \text{ rpm Clockwise}$$

$$\omega_{OA} = \frac{2\pi \times 120}{60} = 4\pi \frac{\text{Rad}}{\text{Sec}}$$

$$V_A = 0.2 \times 4\pi = 2.5132 \text{ m/s}$$

link 3, (A, B) (I_{13}) wrt ground (Absolute vel. obtain ω wrt to ground)

$$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13A}} = \frac{V_B}{I_{13B}}$$

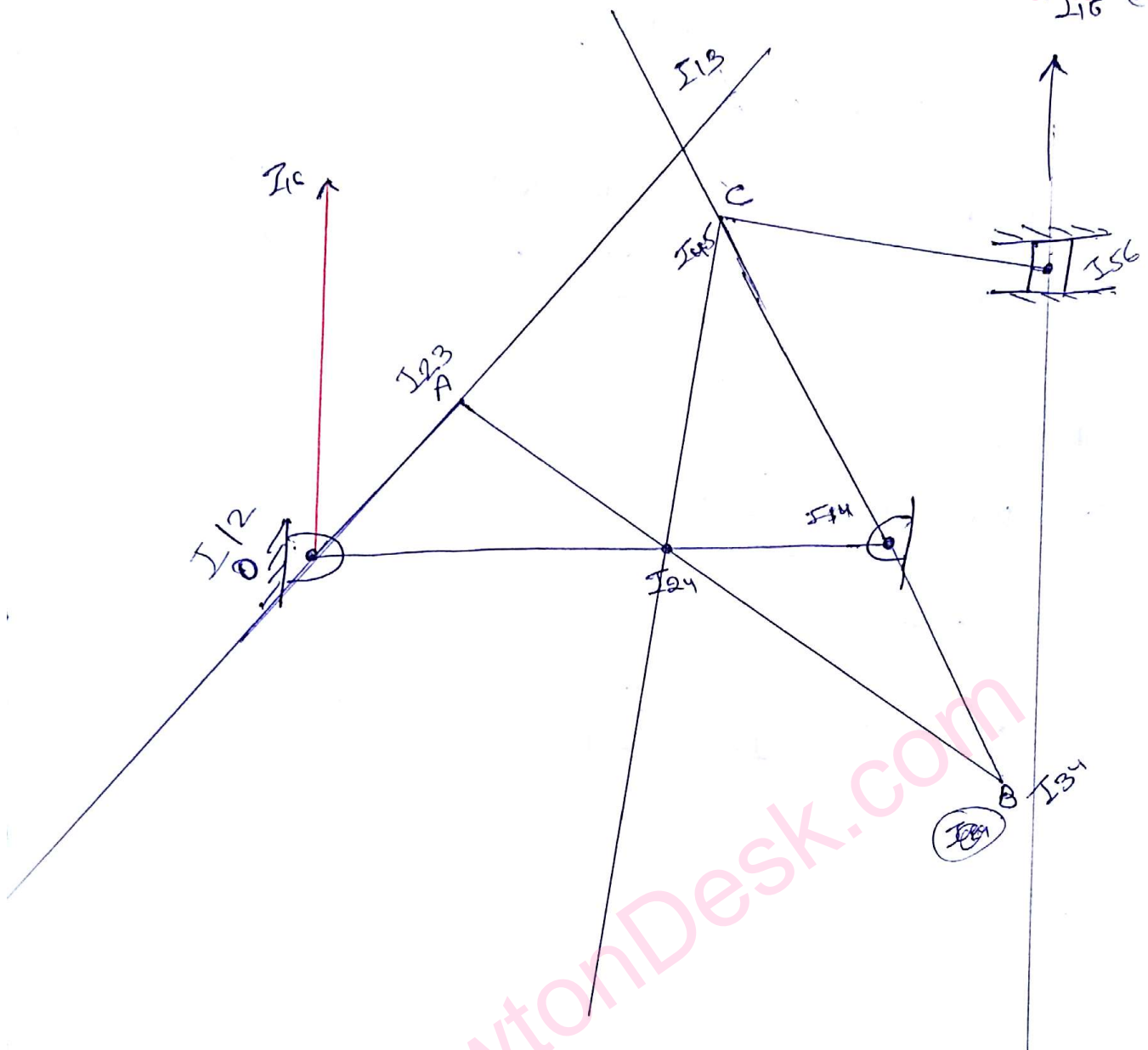
link 4 (B, C) (I_{14})

$$\omega_4 = \omega_{BC} = \frac{V_B}{I_{14B}} = \frac{V_C}{I_{14C}}$$

link 5 (C, D) (I_{15}).

$$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15C}} = \frac{V_D}{I_{15D}}$$

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Kennedy's Theorem :

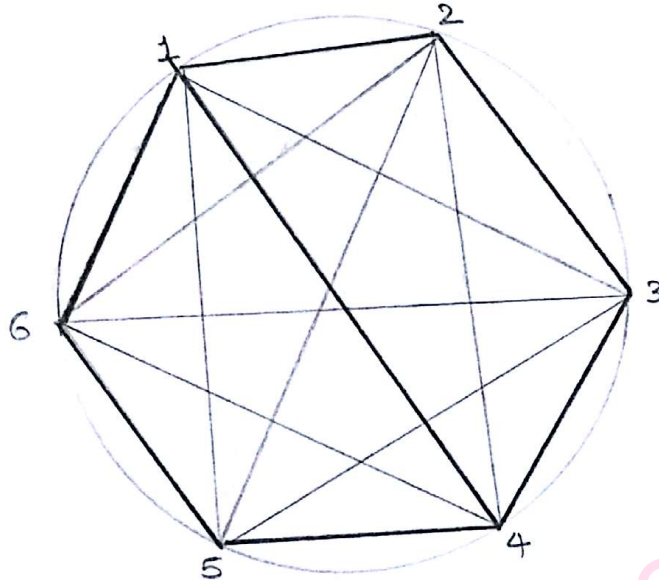
→ For the Relative motion b/w the NO. of links in a Mechanism any three links, their three Instantaneous ^{Centres} must lie in straight line.

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Given links = 6

$$IC = \frac{6(6-1)}{2} = \frac{6 \times 5}{2} = \frac{30}{2} = 15$$

$$IC = 15$$



- I₁₂ I₁₃ I₁₄ I₁₅ I₁₆
- I₂₃ I₂₄ I₂₅ I₂₆
- I₃₄ I₃₅ I₃₆
- I₄₅ I₄₆
- I₅₆

$$\textcircled{1} \omega_3 = \omega_{AB} = \frac{V_A}{r_{BA}} = \frac{V_B}{r_{AB}} \Rightarrow V_B = \frac{2.5132 \times 1.1}{0.8} = 3.455 \text{ m/s}$$

$$\omega_{AB} = \frac{2.5132}{0.8} = 3.14 \text{ rad/s}$$

$$\textcircled{2} \omega_4 = \omega_{BC} = \frac{V_B}{r_{CB}} = \frac{V_C}{r_{BC}} \Rightarrow V_C = \frac{3.455 \times 1.2}{1.4} = 1.727 \text{ m/s}$$

$$\omega_{BC} = \frac{3.455}{1.4} = 2.467 \text{ rad/s}$$

$$\omega_5 = \omega_{CD} = \frac{V_C}{r_{CD}} = \frac{V_D}{r_{DC}} \Rightarrow V_D = \frac{1.727 \times 1.5}{1.7} = 1.11 \text{ m/s}$$

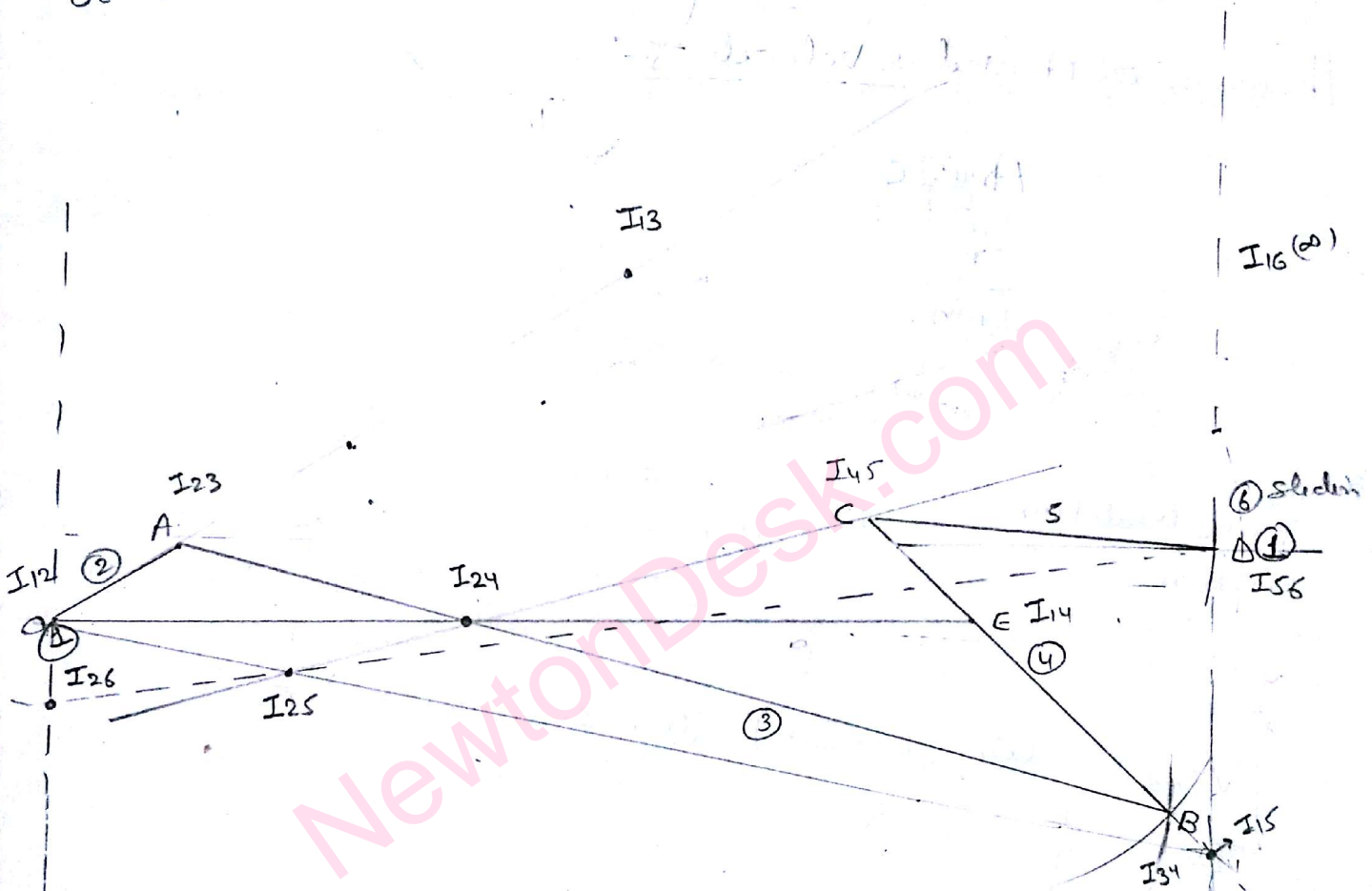
$$\omega_D = \frac{1.11}{1.7} = 0.65 \text{ rad/s}$$

- V_B = 3.2 m/s
- V_C = 1.6 m/s
- V_D = 1.08 m/s
- ω_{AB} = 2.99
- ω_{BC} = 2.467
- ω_{CD} = 2.16

GRAPHICAL Representation

Scale 1cm = 100mm

- AB = 1.5 m → 1500 mm → 15 cm
- BC = 600 mm → 6 cm
- CD = 500 mm → 5 cm
- BE = 400 mm → 4 cm
- OE = 1.35 m → 1350 mm → 13.5 cm.



- $I_{13A} = 8 \text{ cm} \rightarrow 800 \text{ mm} = 0.8 \text{ m}$
- $I_{13B} = 11 \text{ cm} \rightarrow 1100 \text{ mm} = 1.1 \text{ m}$
- $I_{14B} = 4 \text{ cm} \rightarrow 400 \text{ mm} = 0.4 \text{ m}$
- $I_{14C} = 2 \text{ cm} \rightarrow 200 \text{ mm} = 0.2 \text{ m}$
- $I_{15C} = 7 \text{ cm} \rightarrow 700 \text{ mm} = 0.7 \text{ m}$
- $I_{15D} = 4.5 \text{ cm} \rightarrow 450 \text{ mm} = 0.45 \text{ m}$

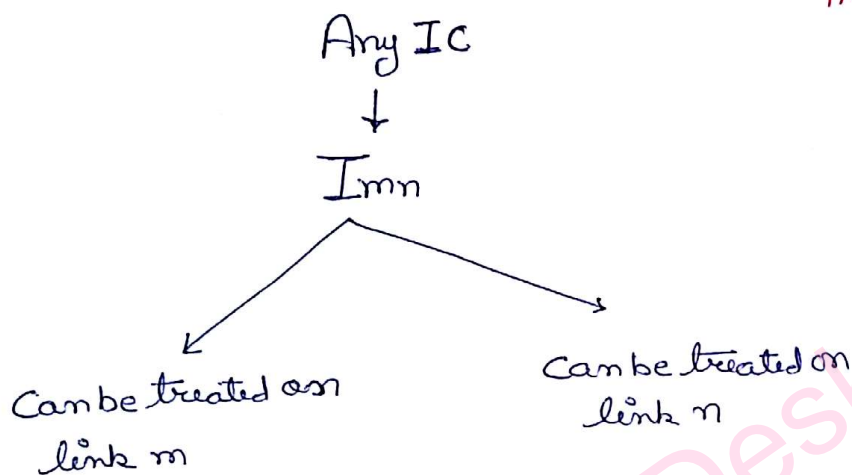
Kennedy's Theorem : only for Absolute Results

For the Relative motion b/w the no. of links in a Mechanism, Any three links, their three instantaneous Centres must lie in a Straight line.

Theorem of Angular velocities :-

This Theorem is applied at

I_{mn}



$$V_{I_{mn}} = \omega_m (I_{mn} I_m) = (\omega_m I_{mn} I_m) = V_{I_{mn}}$$

But total IC in case :-

I_{mn} - link 1

I_{lm} - link 2

I_{ln} - link 3

If I_{lm}, I_{ln} lie at the same side of I_{mn} → Dirⁿ will be same.

If I_{lm}, I_{ln} don't lie at same side of I_{mn} → Dirⁿ will be different.

$\omega_2 =$ Given in previous question
(clockwise) at (2) link

$I_{25}:-$

$$\omega_2 (I_{25} I_{12}) = \omega_5 (I_{25} I_{15})$$

Distance → (A.C)

$I_{24}:-$

$$\omega_2 (I_{24} I_{12}) = \omega_4 (I_{24} I_{14})$$

→ AC

$I_{45}:-$

$$\omega_4 (I_{45} I_{14}) = \omega_5 (I_{45} I_{15})$$

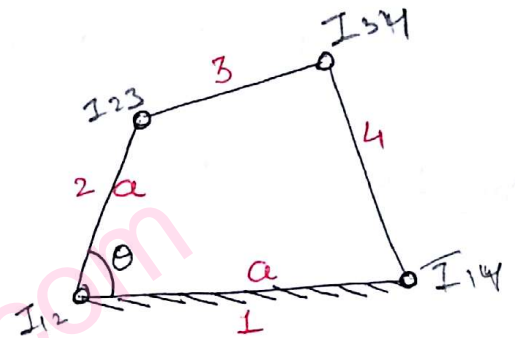
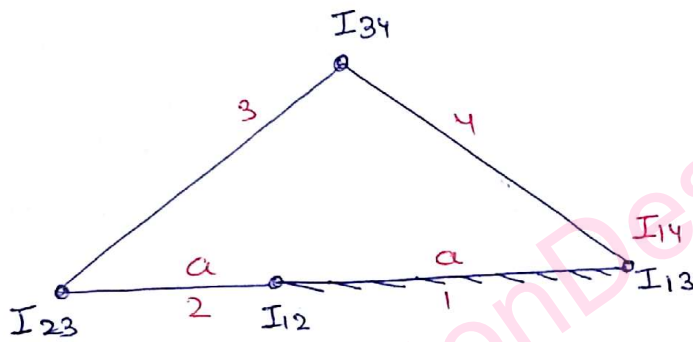
→ AC

Problem 1

When $\theta = 180^\circ$

$\omega_2 = 5 \text{ rad/s}$

$\omega_3 = ?$

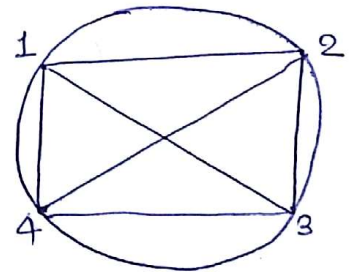


$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$5(a) = \omega_3 (2a)$$

$\omega_3 = 5 \text{ rad/sec}$

Ans



$$13 \begin{cases} 12, 23 \\ 14, 43 \end{cases}$$

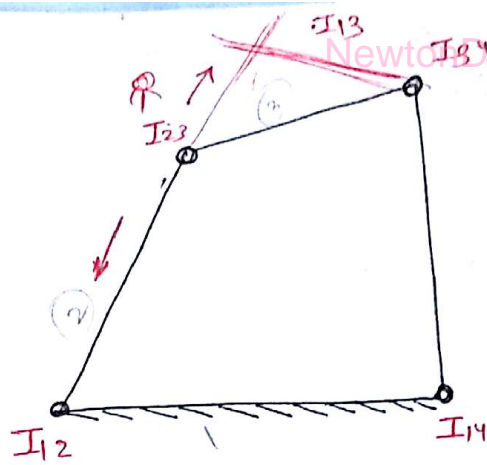
$$24 \begin{cases} 23, 34 \\ 21, 14 \end{cases}$$

$\omega_2 = 5 \text{ rad/s (clock)}$

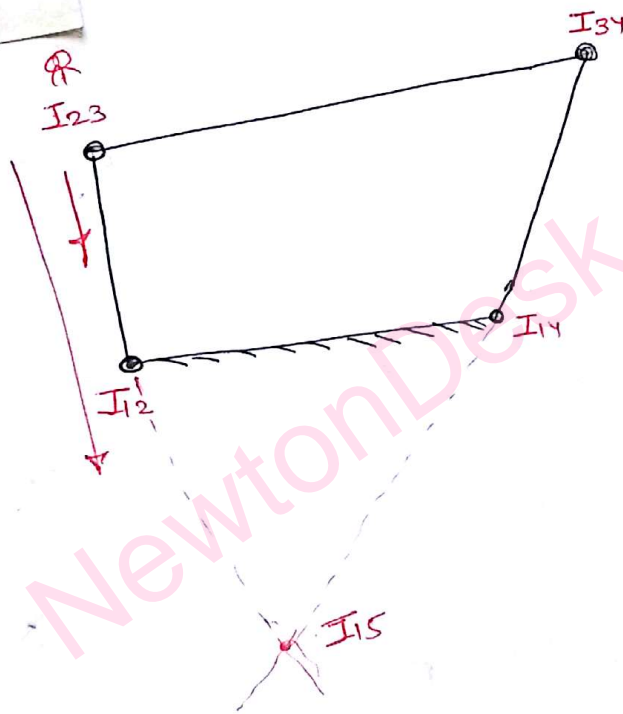
$\omega_3 = 14 \text{ rad/s}$

find the angular velocity of link 2
w.r.to link 3 **clockwise (+)**

$$\begin{aligned}\vec{\omega}_{23} &= \vec{\omega}_2 - \vec{\omega}_3 \\ &= (+5) - (-14) \\ &= 5 + 14 \\ &= 19 \text{ rad/sec. clock.}\end{aligned}$$



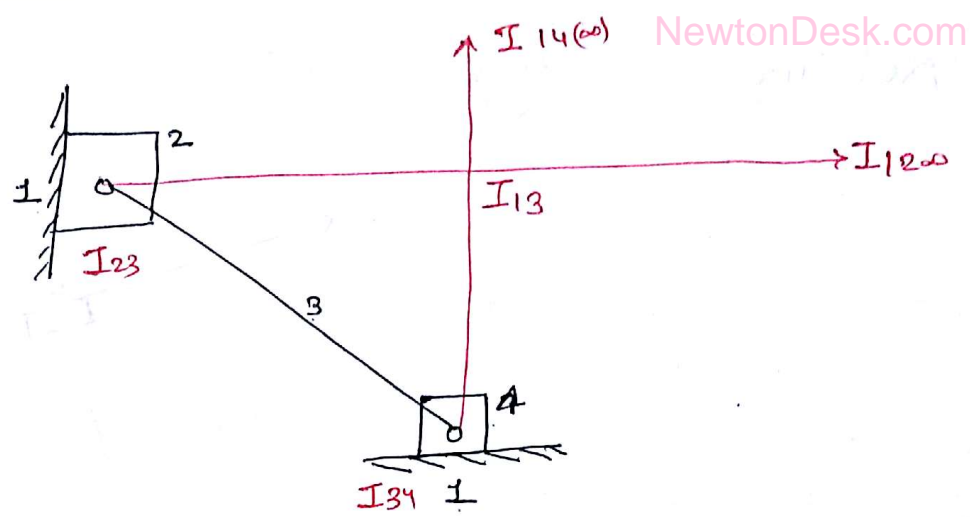
Qm)



$$\begin{aligned}\vec{\omega}_{23} &= +5 - (-14) \\ &= -9 \\ \vec{\omega}_{23} &= -9 \text{ Rad/sec} \\ \vec{\omega}_{23} &= 9 \text{ rad/sec (Ac)}\end{aligned}$$

P.D. rk

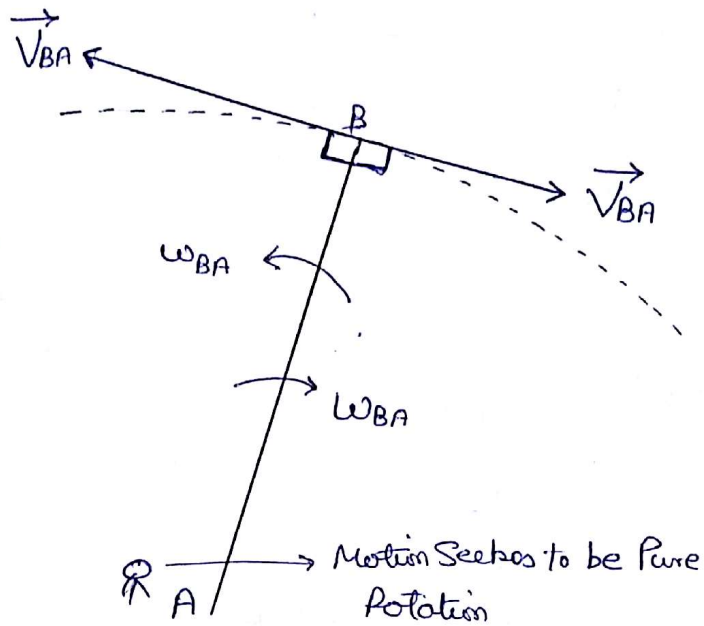
Velocity is I_{24} ?



24
 will
 be at
 (∞)
 Any
 $23, 24$
 $21, 24$
 $(\infty)(\infty)$
 \downarrow
~~only~~
 infinitely only displaced
 line

Relative Velocity Method :-

The Velocity of Point B,
w.r.to. point A
will be in the dirⁿ
⊥ to the link AB.



Point	w.r.to	Procedure
A	O	Plot link ⊥ to link OA
B	A	Plot link ⊥ to link AB
B	E	Plot link ⊥ to link EB
C	$BE/BE = bc/be$	bc/be
D	C	line ⊥ to link CD
D	fixed	line to the motion of slider

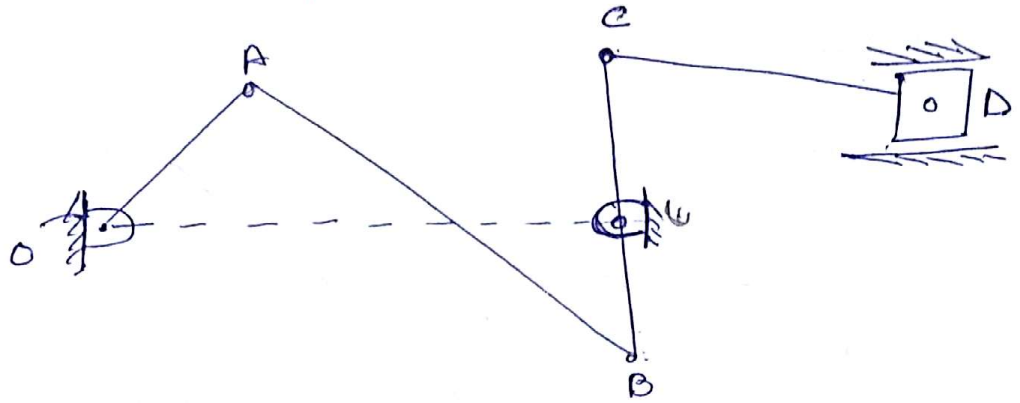
$$\omega_{AB} = \frac{V_{AB}}{AB} \text{ Rad/s (AC)}$$

$$\omega_{BC} = \frac{V_{BC}}{BC} \text{ Rad/s (AC)}$$

$$\omega_{CD} = \frac{V_{CD}}{CD} \text{ Rad/s (AC)}$$

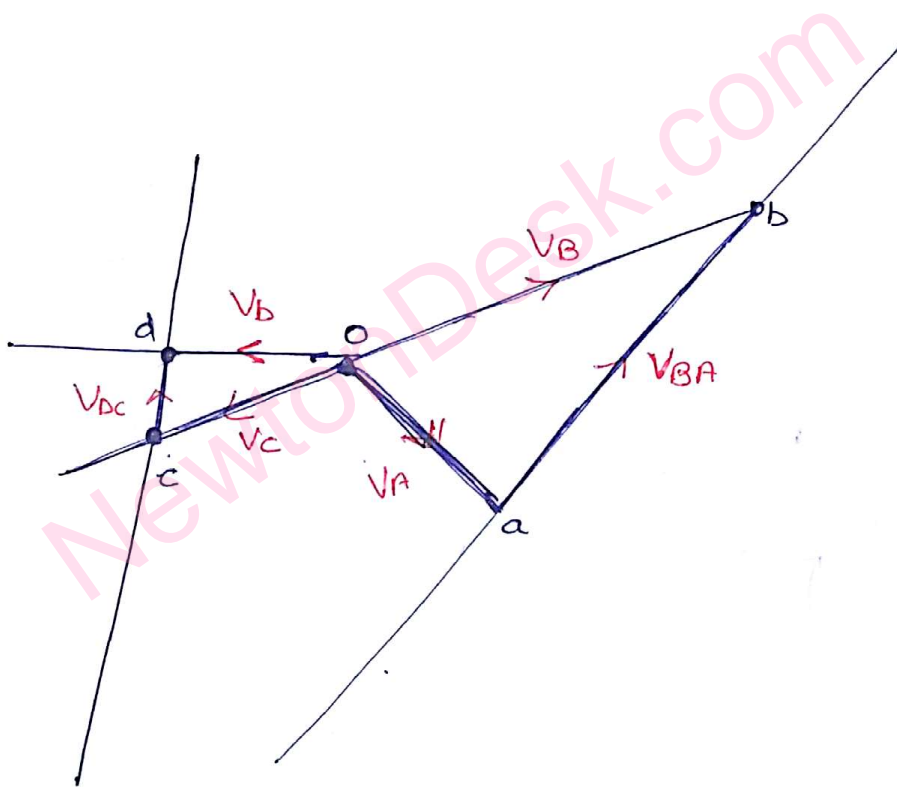
Graphical
Scad.

All letters in Configuration dia. will be Capital



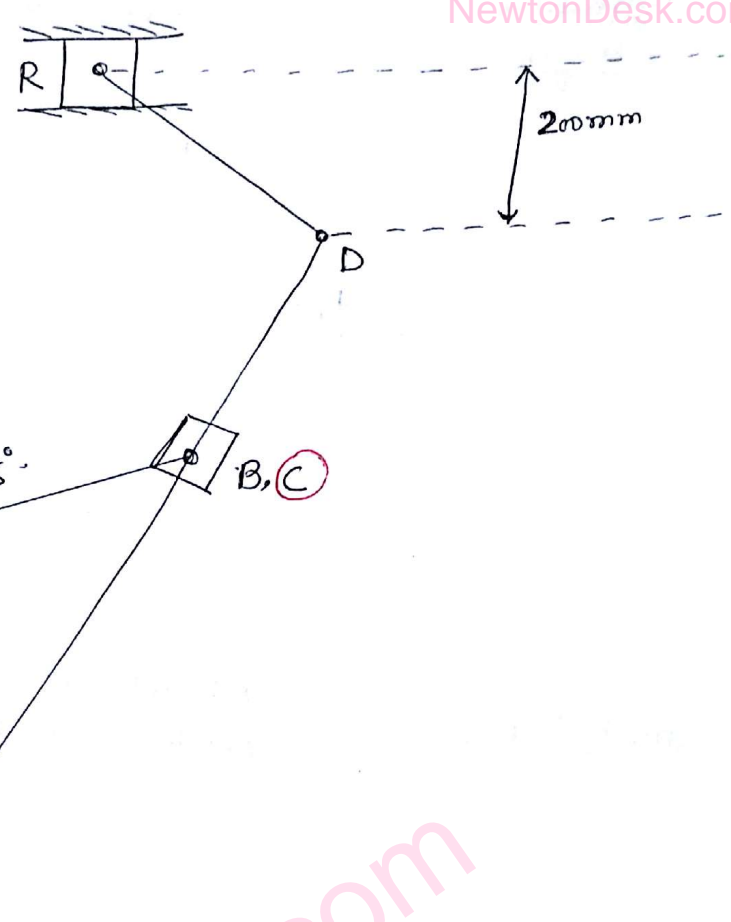
Velocity Profile

Absolute → motion with respect to fixed
Relative → motion w.r. to another



Pb.

of Crank O₁B
 ↳ 40 rpm
 (AC)



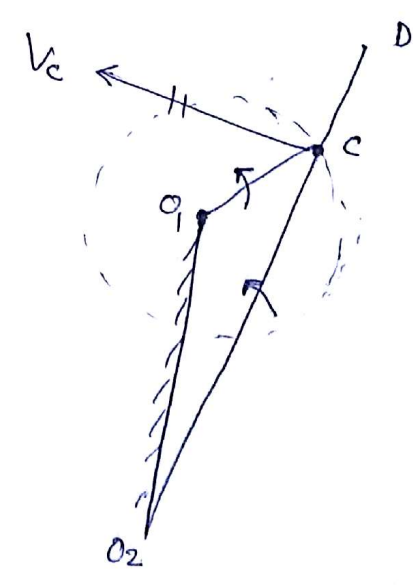
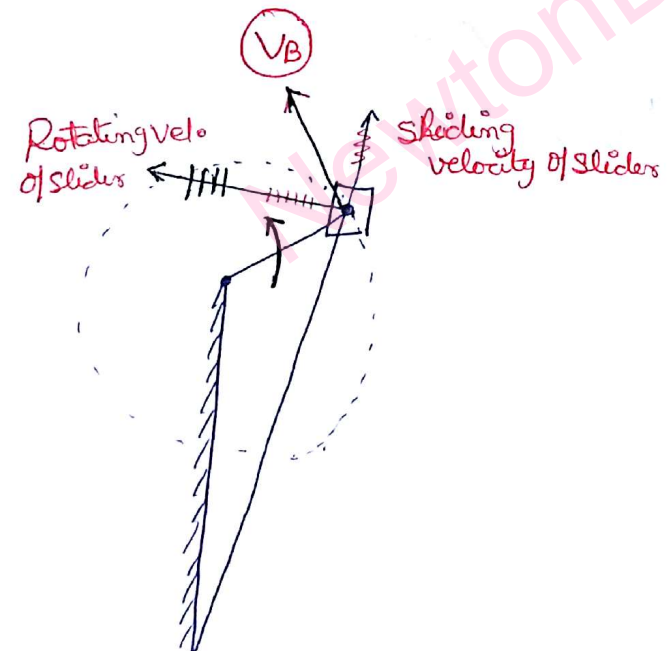
Given data

$$V_B = \omega_{O_1B} \times r_{O_1B}$$

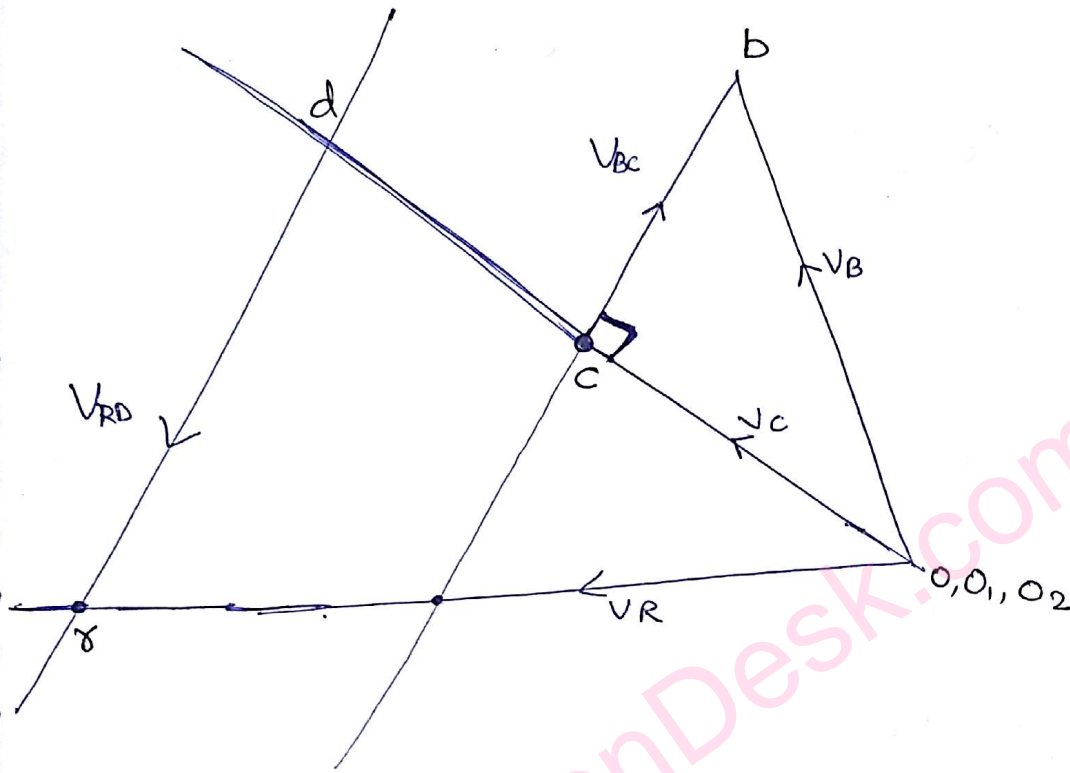
$$V_B = 0.25 \times \frac{2\pi \times 40}{60} \text{ m/s}$$

Scale of velocity

Point C
 ↓
 Coincident Point of
 Slider B, But on
 slotted bar.
 Slotted bar → O₂CD



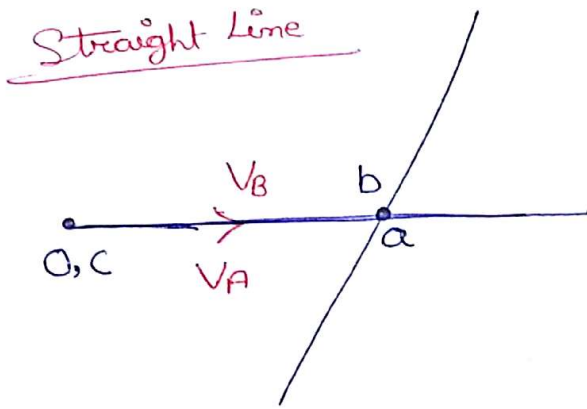
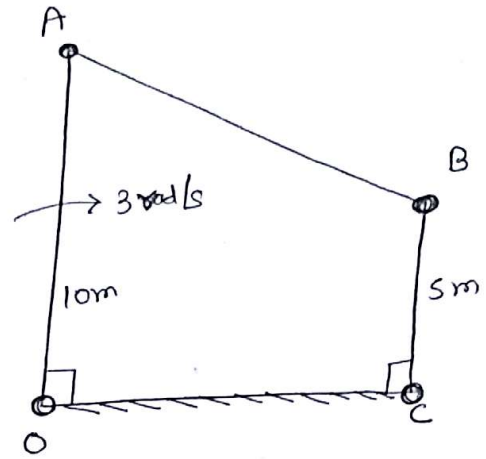
Point	w.r.t	Procedure
C	O_2	line L to link O_2C
C	B	line l to link D_2C



NewtonDesk.com

Pb. 2) (2M)

The velocity diagram for the given Configⁿ at this instant will be:-



Pb. 2) $V_B = ?$

(2M) $V_B = V_A = 10 \times 3 = 30 \text{ m/s}$

Pb. 3) $\vec{V}_{BA} = ?$

Zero

$V_B = V_A$

Pb. 4) The motion of link AB at this instant will be,

$\omega_{AB} = \frac{V_{AB}^{\circ}}{AB} = 0$
 → Pure Translation

Pb. 5) $\omega_{BC} = ?$ *

$V_B = V_A$

$5 \times \omega_{BC} = 10 \times 3 \Rightarrow \omega_{BC} = \frac{30}{5} = 6 \text{ rad/sec.}$

Q. 6 (2M)
 in

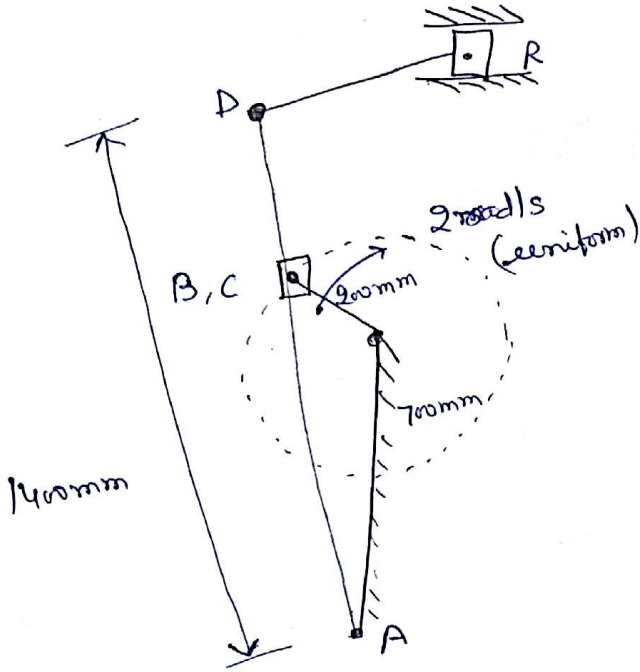
- 1 _____
- 2 _____
- 3 _____
- 4 _____
- 5 _____

- a) 1, 2, 3
- b) 2, 4
- c) 1, 3
- d) -- NOT.

B

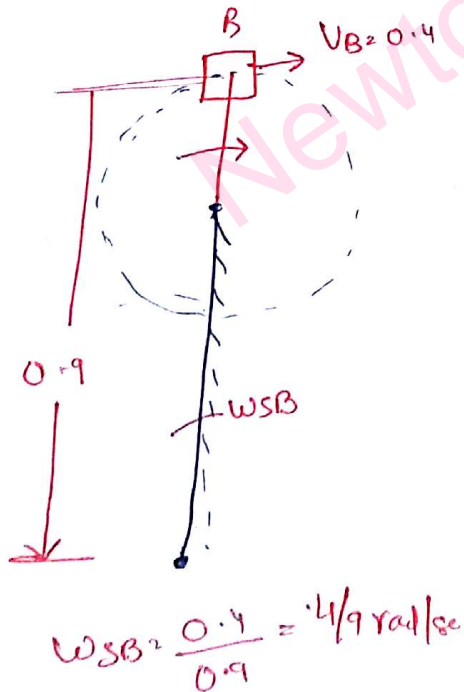
$$V_B = 0.2 \times 2 = 0.4 \text{ m/s.}$$

(uniform)

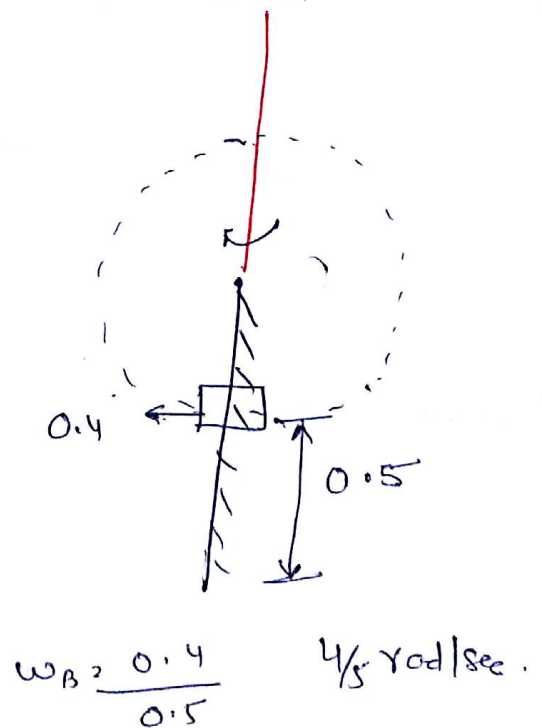


find the Avg. Velocity of Slotted Base, when vel. of RAM is Max.

Cutting Stroke



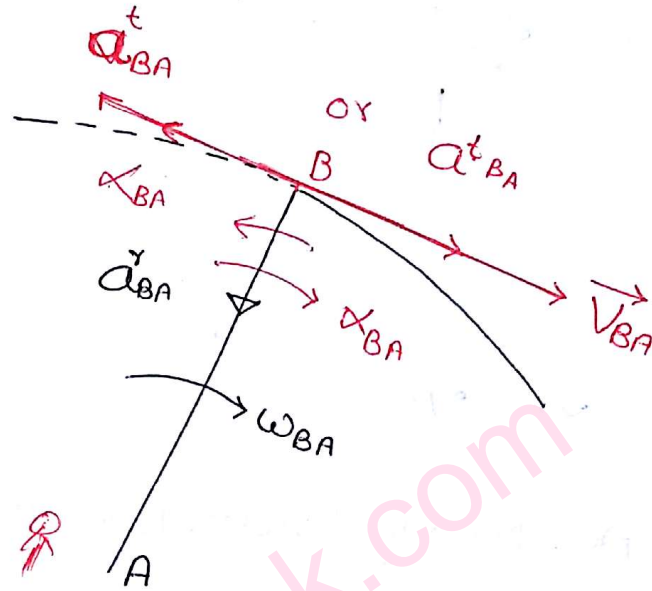
Return Stroke



Acceleration Diagrams

(only objective)

Acceleration Analysis



Because of the change in dirⁿ of velocity

$$\vec{a}_{BA} = \frac{d}{dt} \vec{V}_{BA}$$

$$a_{BA}^r = \frac{V_{BA}^2}{BA} \quad (B \rightarrow A)$$

Known for all link (find via vel. dia)

$$a_{BA}^t = (BA) \omega_{BA}$$

Bez of change in magnitude of velocity.

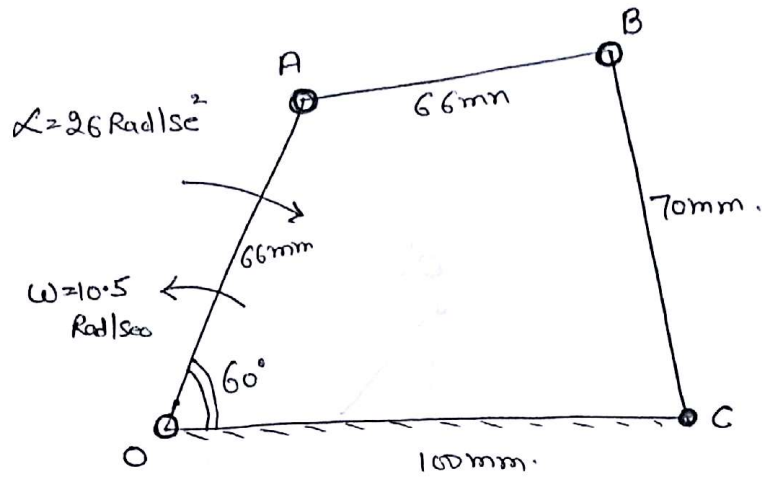
May or May not be zero

→ only for known input link

→ ⊥ to the Radial dirⁿ

→ not mentioned in ques assumed to be 0.

(Pn)



find $\rightarrow a_B = ?$ $\alpha_{BA} = ?$
 $a_{BA} = ?$ $\alpha_{BC} = ?$

(ans)

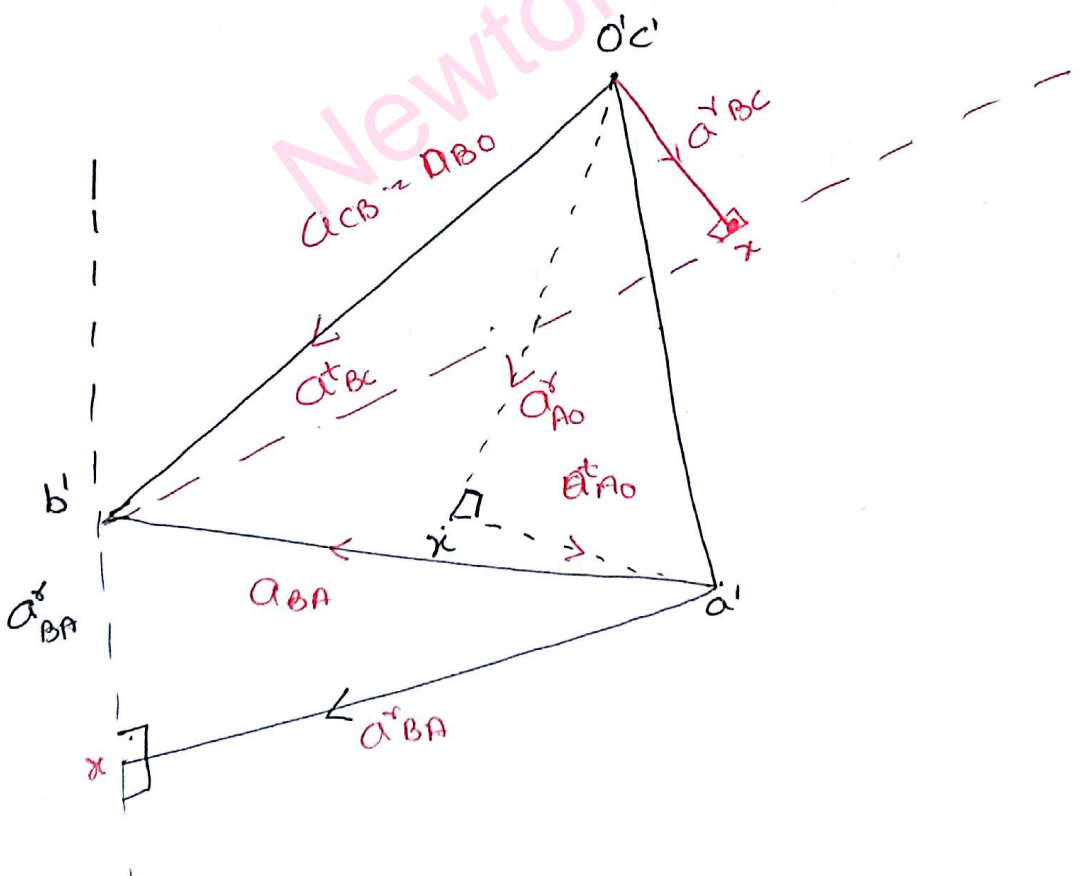
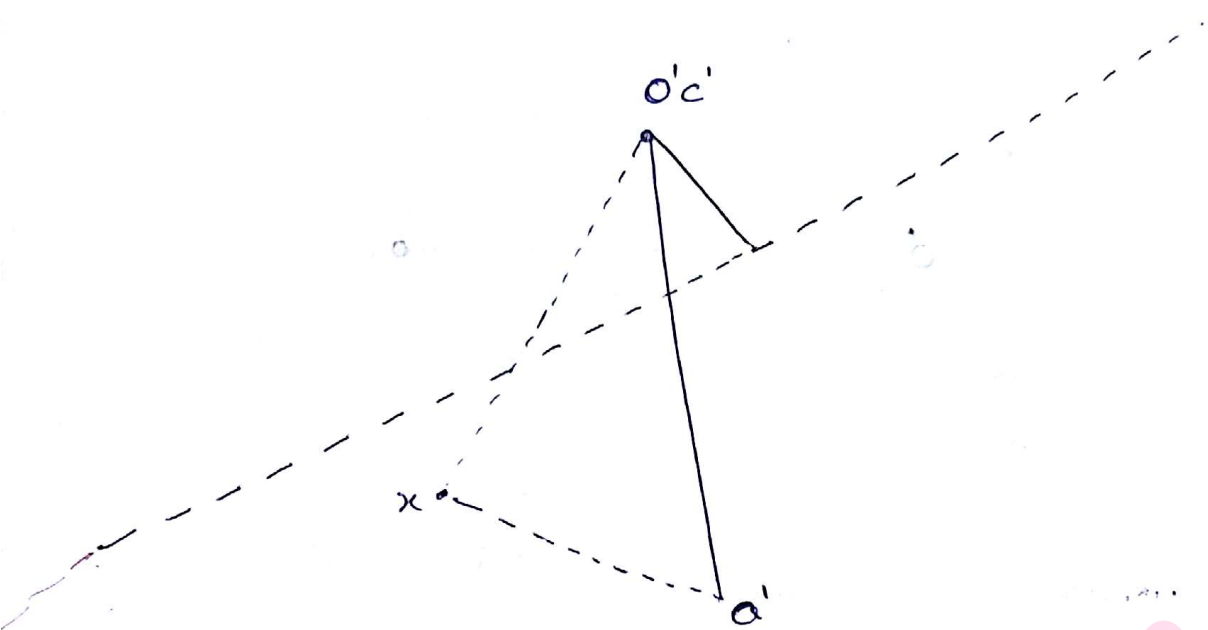
Point	w.r.to	Procedure
A	O	$a_{AO}^y = \frac{V_{AO}^2}{AO}$ (A \rightarrow O) (known) $a_{AO}^t = (AO) \alpha_{AO}$ (known) (\perp to dir ⁿ of Radial)
B	A	$a_{BA}^y = \frac{V_{BA}^2}{BA}$ (B \rightarrow A) $a_{BA}^t = (BA) \alpha_{BA}$ (Unknown)
C	B	$a_{BC}^y = \frac{V_{BC}^2}{BC}$ (B \rightarrow C) (known) $a_{BC}^t = (BC) \alpha_{BC}$ (unknown) \perp to dir ⁿ of Radial

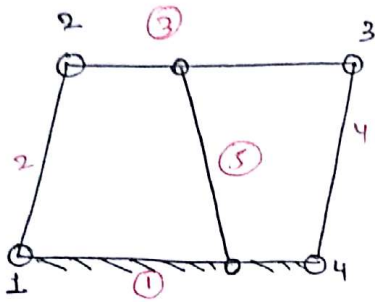
$$a_{AO}^y = \frac{V_{AO}^2}{AO}$$

$$a_{BA}^y = \frac{V_{BA}^2}{BA}$$

$$a_{BC}^y = \frac{V_{BC}^2}{BC}$$

$$a_{(AO)}^t = (AO) \alpha_{AO}$$



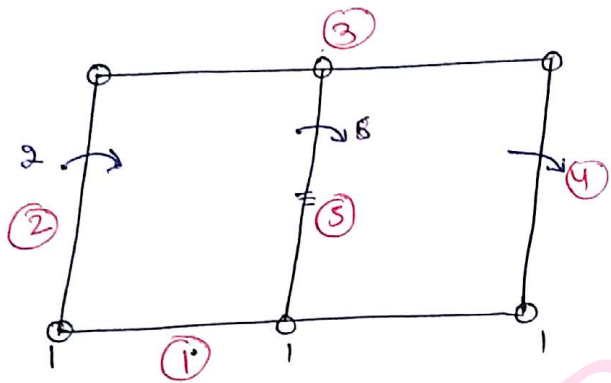


$l = 5$
 $j = 6$
 $h = 0$

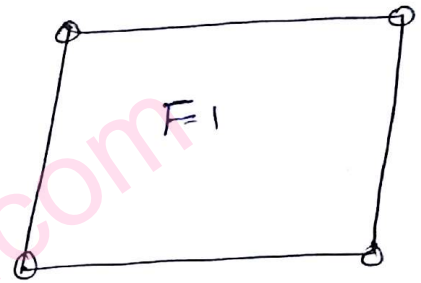
$F = 3(5-1) - 2 \times 6$

$F = 0$

frame structure



\equiv



$F = 1$

$l = 5$
 $j = 6$
 $h = 0$

$F = 3(5-1) - 2 \times 6 = 0$

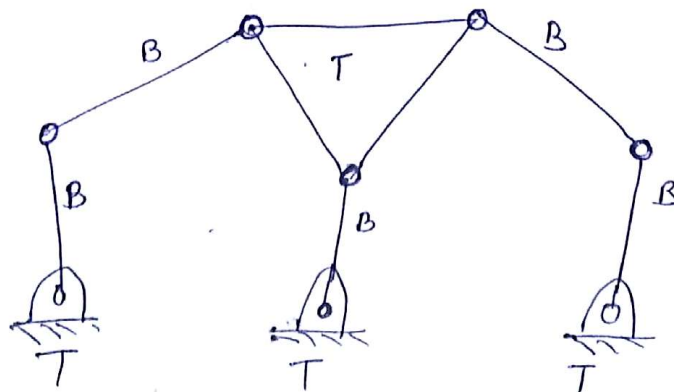
$F = 0$

$F = 1$ * *

Binary link \rightarrow Connected at two places

Ternary link \rightarrow Connected at three places

Quaternary link \rightarrow Connected at four places



KLEIN'S CONSTRUCTION

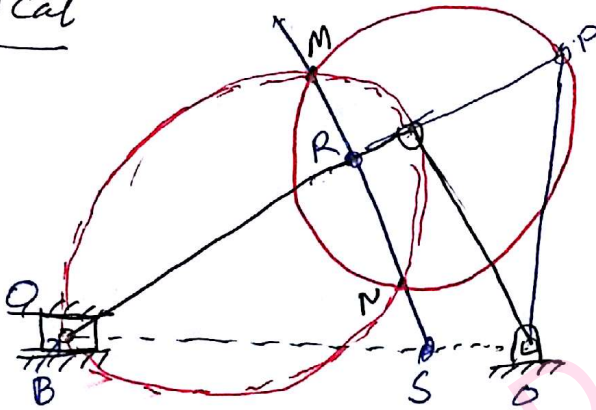
"It is only applied in single slider crank mechanism (BASIC)"

When $\alpha_{crank} = 0$

Given ω_{crank}

Graphical

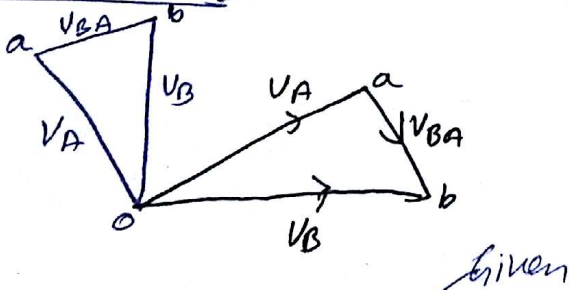
Scale



ΔOAP

Velocity Δ

Velocity Dig (Rough)

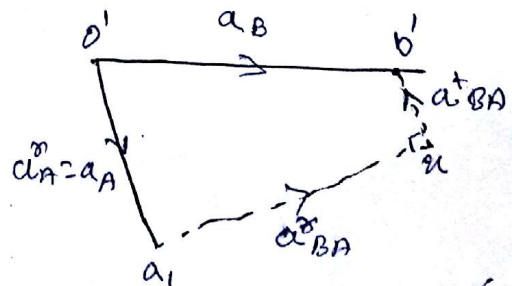


$$\frac{VA}{OA} = \frac{VB}{OB} = \frac{VBA}{AB} = \omega_{crank}$$

$\Delta OARS$

Accⁿ Δ

Accⁿ Dia (Rough)



$$\frac{aa}{OA} = \frac{aBA}{RS} = \frac{ab}{OS} = \frac{aBA}{AB} = \omega_{crank}^2$$

Coriolis Accⁿ (a^c):-

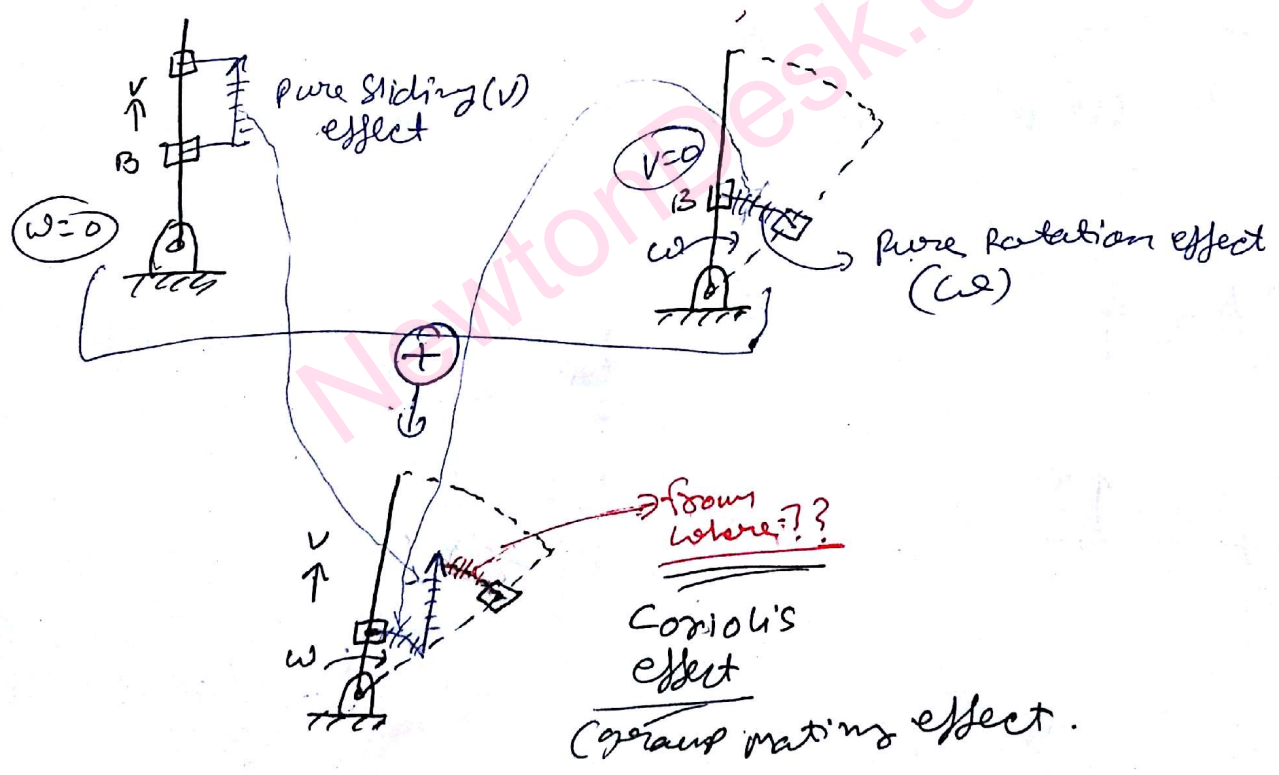
∴ This accⁿ will always be associated with the slider when the slider is sliding on the rotating body.

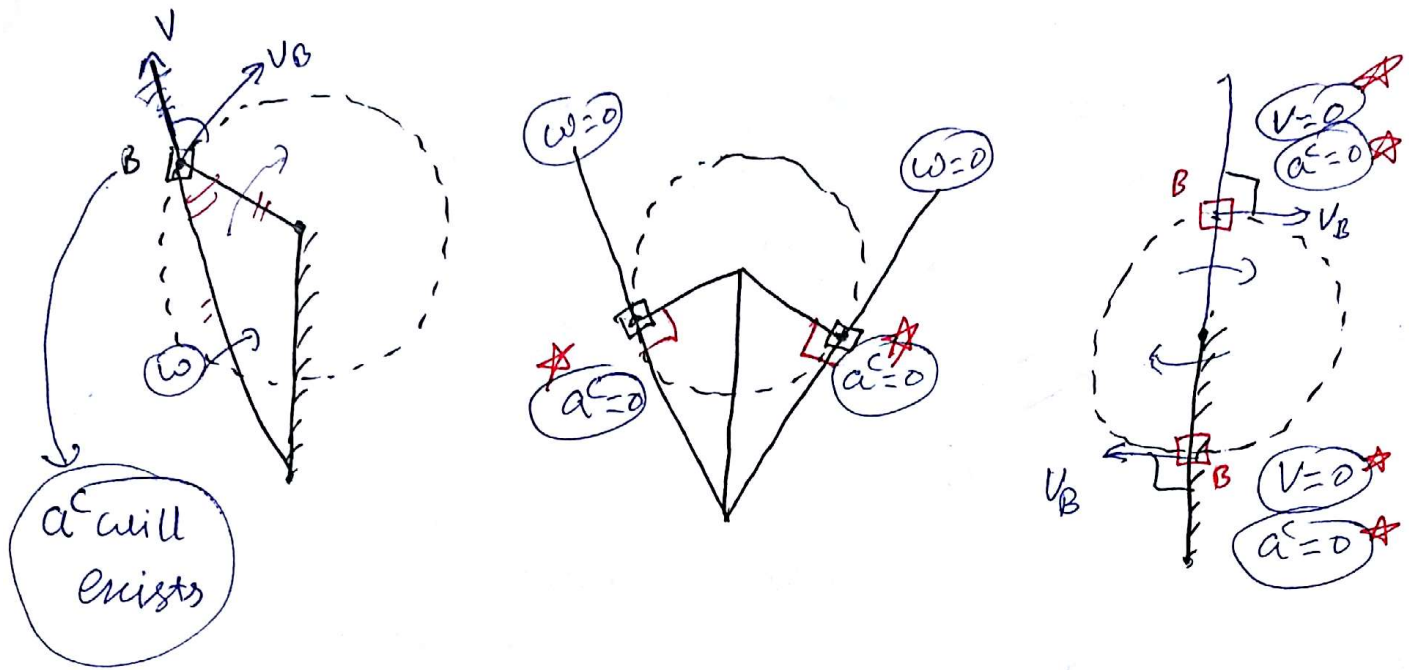
The magnitude is (a^c)

$$a^c = 2v\omega$$

v ⇒ Sliding vel. of slider

ω → Ang. vel. of body on which slider is sliding.

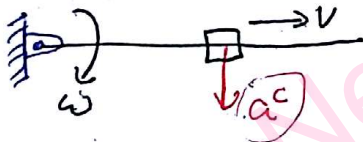




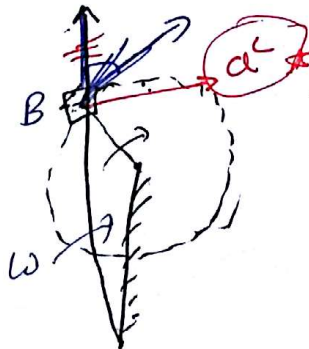
Direction of (a^c) :

- (i) Take the sense of ω .
- (ii) Rotate the \vec{v} in that sense by 90° .

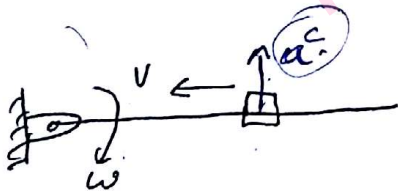
1)



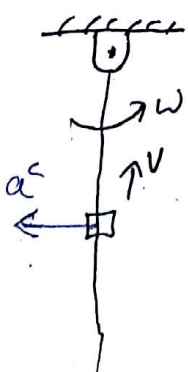
5)



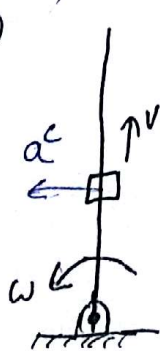
2)



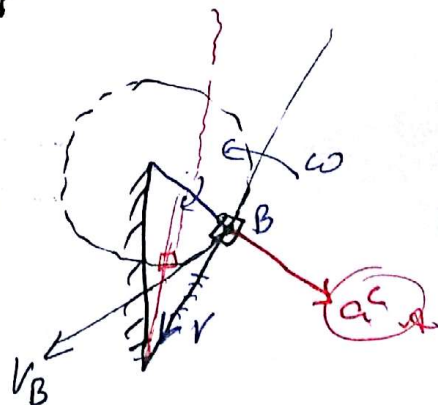
3)

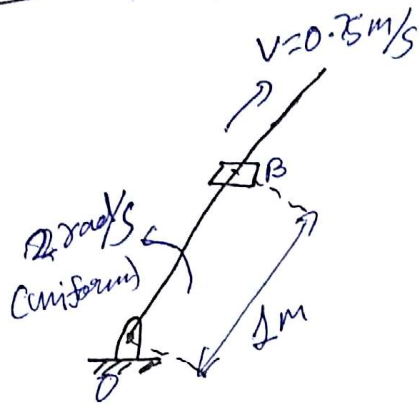


4)



6)



Prob

Find the Accⁿ of slider $a_B = ?$

$$\omega = 2\text{ rad/s}$$

$$v = 0.75\text{ m/s}$$

 $\frac{1}{2}$

$$2v\omega$$

$$a_B^c = 2v\omega$$

$$= \underline{\underline{3\text{ m/s}^2}}$$

Diagram showing the acceleration components of the slider B . The centripetal acceleration a_B^c is directed towards the pivot A (to the left). The tangential acceleration a_B^o is directed downwards. The total acceleration a_B is the vector sum of these two components.

$$a_B^o = r\omega^2$$

$$= (1)(2)^2$$

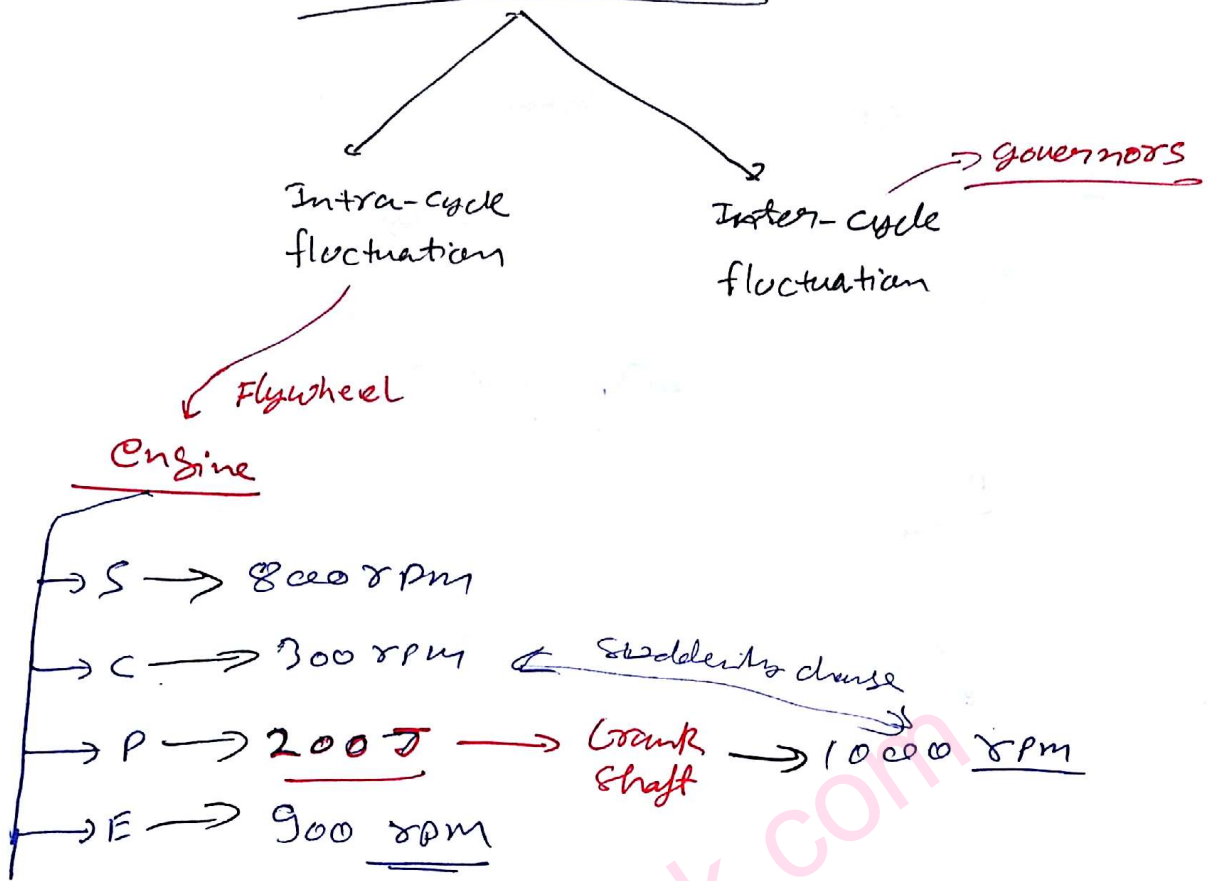
$$= \underline{\underline{4\text{ m/s}^2}}$$

$$a_B = \sqrt{(3)^2 + (4)^2}$$

$$= \underline{\underline{5\text{ m/s}^2}}$$

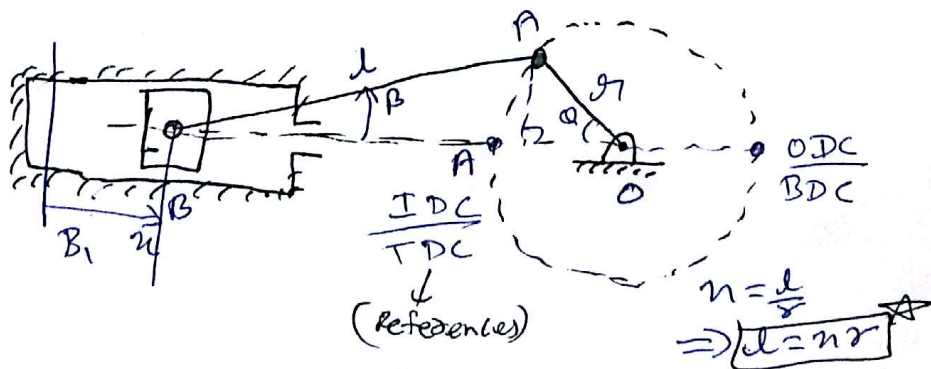
INSTANTANEOUS FLUCTUATION OF SPEED

CONTROL DEVICES :-



Kinematic Analysis of Single Slider Crank Mechanism :-

(Inertia of connecting rod is not considered in this analysis for being the lightest body)



$$Am \left\{ \begin{aligned} l \sin \beta &= r \sin \alpha \\ \sin \beta &= \frac{\sin \alpha}{n} \end{aligned} \right.$$

$$\cos \beta = \frac{\sqrt{1 - \sin^2 \theta}}{n} = \frac{\sqrt{n^2 - \sin^2 \theta}}{n}$$

$m \rightarrow$ Mass of Reciprocating Parts.

$l \rightarrow$ length of Connecting rod (C.R)

$r =$ radius of Crank.

$$n = \frac{l}{r} \Rightarrow \frac{\text{connecting rod length}}{\text{Crank radius}}$$

$\omega \Rightarrow$ Ang. vel. of Crank

$\theta \rightarrow$ Angle turned by Crank from IDC/TDC

$$\omega = \frac{d\theta}{dt}$$

PISTON MOTION

Displacement

$$x = B_1 B$$

$$= B_1 O - B O$$

$$= (r+l) - (Bm + m_0)$$

$$= (r+l) - (l \cos \beta + r \cos \theta)$$

$$= r + nr - nr \cdot \frac{\sqrt{n^2 - \sin^2 \theta}}{n} - r \cos \theta$$

$$= r(1 - \cos \theta) + r(n - \sqrt{n^2 - \sin^2 \theta})$$

$$x = r \left[(1 - \cos \theta) + (n - \sqrt{n^2 - \sin^2 \theta}) \right]$$

VELOCITY (V)

$$V = \frac{dx}{dt} = \frac{dx}{d\theta} \left(\frac{d\theta}{dt} \right) \rightarrow \omega$$

$$V = r\omega \left[\sin \theta + \frac{\sin^3 \theta}{2\sqrt{n^2 - \sin^2 \theta}} \right]$$

$$v = r\omega \left[\sin\theta + \frac{\sin 2\theta}{2n} \right] \quad (n \text{ large})$$

Accenⁿ(a) :-

$$a = \frac{dv}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} \rightarrow r\omega$$

$$a = r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n} \right] \quad (n \text{ large})$$

Connecting Rod Motion :-

Ang. Velocity (ω_{CR})

$$\omega_{CR} = \frac{d\beta}{dt}$$

$$\sin\beta = \frac{\sin\alpha}{n}$$

Diff. both side

$$\cos\beta \cdot \frac{d\beta}{dt} = \frac{\cos\alpha}{n} \cdot \frac{d\alpha}{dt}$$

$$\omega_{CR} = \frac{\omega \cos\alpha}{n \cdot \frac{\sin\alpha}{\sin\beta}}$$

$$\omega_{CR} = \frac{\omega \cos\alpha}{n}$$

Ang. Accen (α_{CR})

$$\alpha_{CR} = \frac{d\omega_{CR}}{dt} = \frac{d\omega_{CR}}{d\alpha} \cdot \frac{d\alpha}{dt}$$

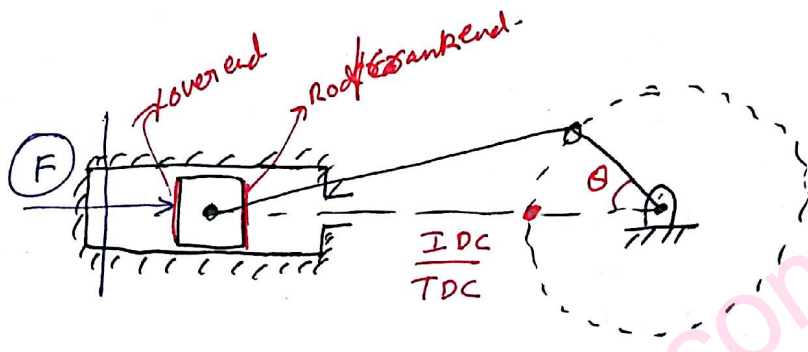
$$\alpha_{CR} = -\frac{\omega^2}{n} \sin\alpha$$

Dynamic analysis of Single slider Crank

Mechanism :-

1) Effective Driving Force to Drive the Piston :-

(Piston Effort) (F)
(Calculated from cover end to Crank end)



Support

Gas pressure
Force

$P_1, P_2 \rightarrow$ Pr. of gas at cover end & Crank end side of piston

$A_1, A_2 \rightarrow$ Cross-sec. areas of piston at cover end & crank end side.

$D \rightarrow$ Piston Dia.

$d \rightarrow$ Piston Rod dia.

$$A_1 = \frac{\pi}{4} D^2$$

$$A_2 = \frac{\pi}{4} (D^2 - d^2)$$



$$F_{gas} = (P_1 A_1 - P_2 A_2)$$

o ppose :-

I) Inertial force

$$F_I = m \cdot a = m \cdot r \omega^2 \left\{ \cos \theta + \frac{\cos^2 \theta}{n} \right\}$$

(ii) kinetic Friction (f)

$$\text{If } \theta \in [0, 180] \rightarrow -f$$

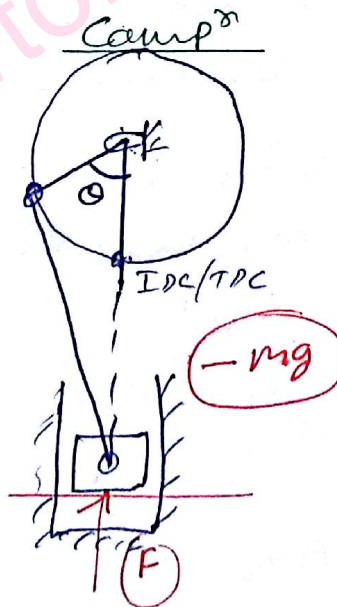
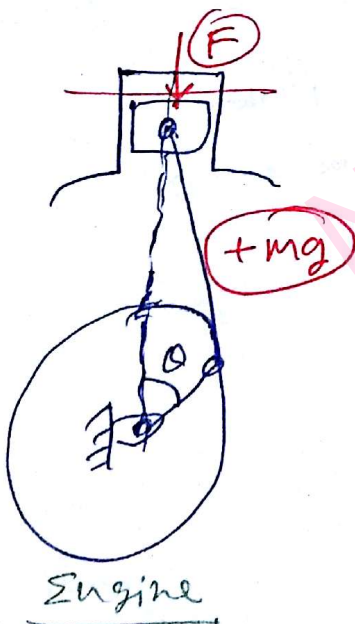
$$\text{If } \theta \in [180, 360] \rightarrow +f$$

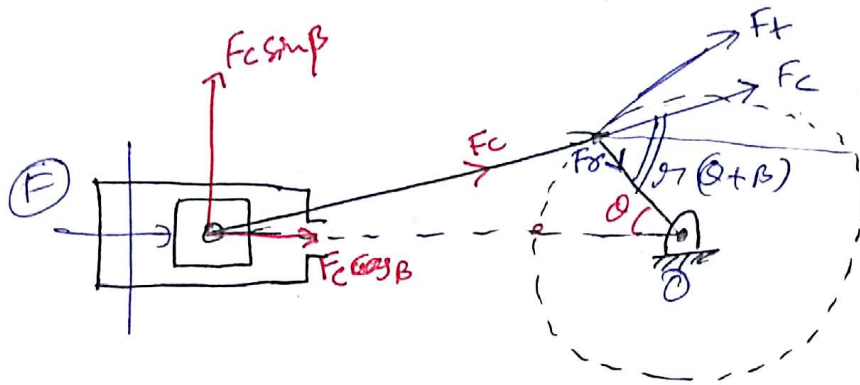
PISTON EFFORT

(Crank end to Crank end)

$$F = (F_{gas} - F_I \pm f) \pm Mg$$

use in case
of only
vertical engine





② Force Along C-R (F_c)

$$F_c \cos B = F$$

$$F_c = \frac{F}{\cos B}$$

③ Normal thrust to Cy. Walls (F_n):-

$$F_n = F_c \sin B$$

$$F_n = F \cdot \tan B$$

④ Crank Effect (F_t)

$$F_t = F_c \sin(\theta + \beta)$$

$$F_t = \frac{F}{\cos B} \sin(\theta + \beta)$$

⑤ Radial Thrust to Crank Shaft Bearings (F_r):-

$$F_r = F_c \cos(\theta + \beta)$$

$$F_r = \frac{F}{\cos B} \cos(\theta + \beta)$$

⑥ Turning Moment on Crank Shaft (T):-

$$T = F_t \cdot r$$

Output of engine

$$T = \frac{F}{\cos \beta} \cdot \sin(\theta + \beta) \cdot r$$

$$T = f(\omega)$$

$$\& \theta = f(\text{time})$$

$$T = F_n(\text{Time})$$

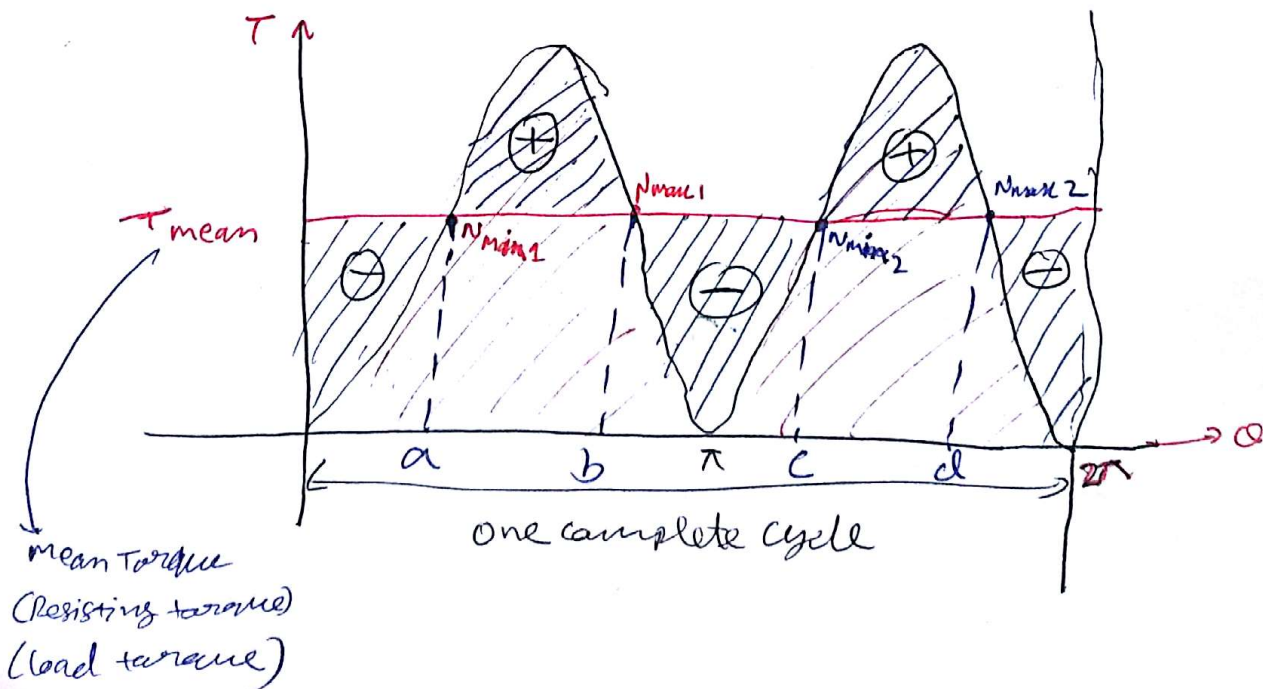
Stress

$$\sigma = F_n(\text{Time})$$

$$\Rightarrow \underline{\alpha = f_n(\text{time})}$$

Fluctuations with jerks

TURNING MOMENT DIAG. OF SINGLE CY. DOUBLE ACTING STEAM ENGINE:-



Area under (T- θ) Dig. in a cycle

$$\Downarrow$$

$$W_{\text{cycle}} = T_{\text{mean}} \times 2\pi$$

$$T_{\text{mean}} = \frac{W_{\text{cycle}}}{2\pi}$$

Even after installing the flywheel:

min ($N_{\text{min}1}, N_{\text{min}2}, \dots$)
 $\hookrightarrow N_{\text{min}}$

Max ($N_{\text{max}1}, N_{\text{max}2}, \dots$)
 $\hookrightarrow N_{\text{max}}$

$$\frac{(N_{\text{max}} - N_{\text{min}})}{N}$$

\hookrightarrow Fluctuations still left

$$\frac{(N_{\text{max}} - N_{\text{min}})}{N} = C_s$$

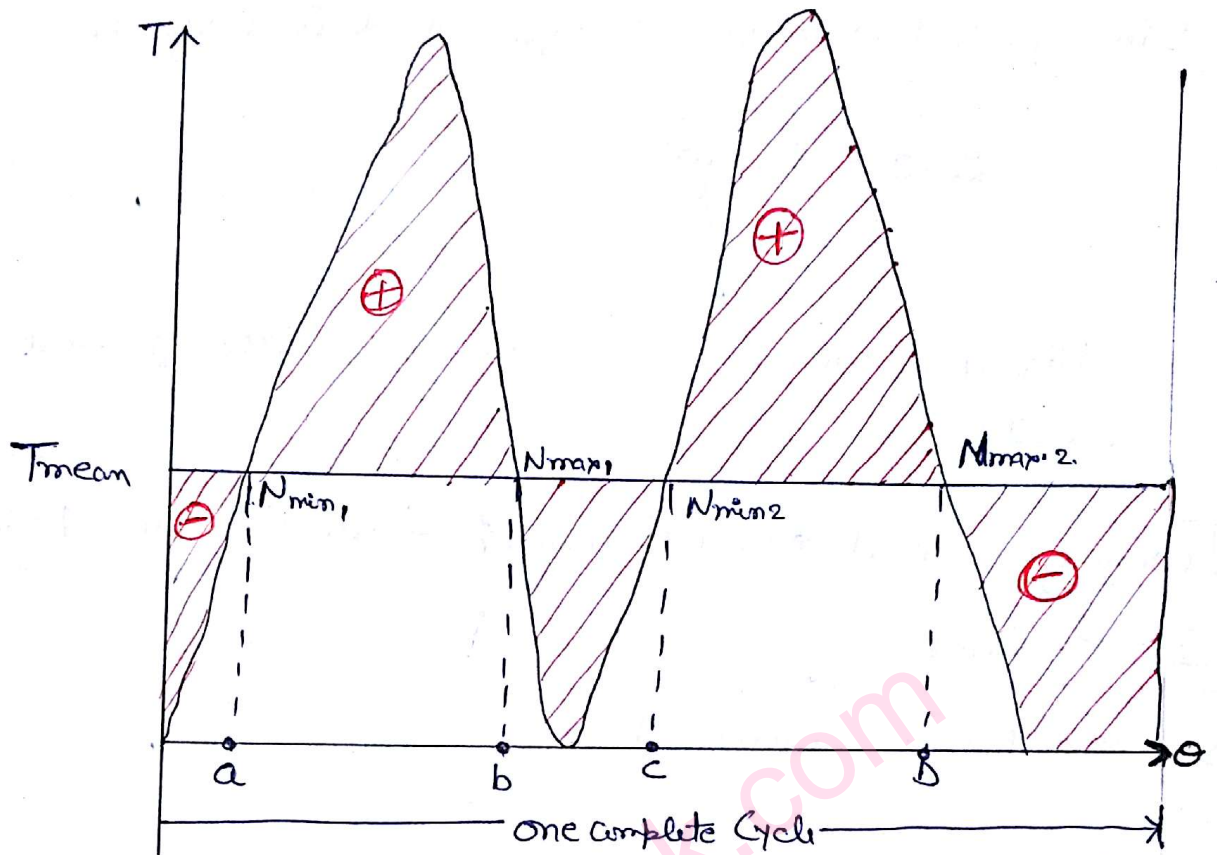
\swarrow
 (mean speed)

\downarrow
 Coefficient of
 fluctuations
 of speed for the
flywheel.

For exam

$$C_s \rightarrow 2\% = 0.02$$

$$\rightarrow \pm 2\% = 4\% = 0.04$$



$$C_E = \frac{(E_{max} - E_{min})}{W_{cycle}}$$

$C_E \rightarrow$ Coefficient of fluctuation of energy for the flywheel.

Any (+ve) or (-ve) area above or below T_{mean} line is Fluctuation of Energy ΔE .

Sum of all the (+ve) area above T_{mean} line
 =
 Sum of all the (-ve) area below T_{mean} line

Main function of flywheel:

→ Storing and delivering energies at desired speed.

e.g. $200 J = \frac{1}{2} I \omega^2$

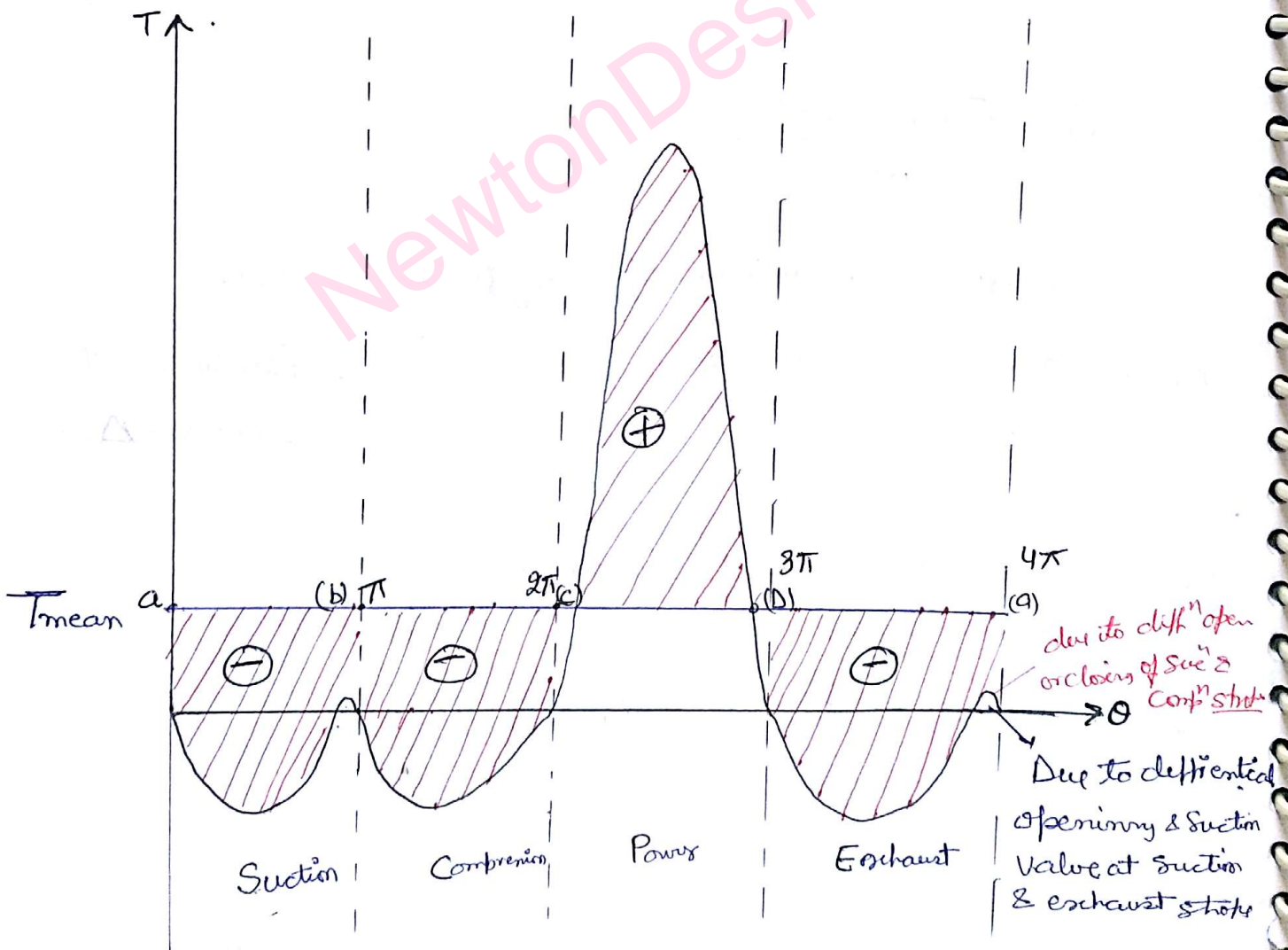
High Speed Running engines are having their flywheel with less I.

Turning Moment form (diagram) of Single Cylinder 4-Stroke

I.C Engine:

$$T_{mean} = \frac{W_{cycle}}{4\pi}$$

$\Delta E \rightarrow$ fluctuation (above & below T_{mean})



Fundamental Equation of Flywheel

$m \rightarrow$ mass of Flywheel

$k \rightarrow$ Radius of gyration

$$\text{Moment of inertia} = \boxed{I = mk^2}$$

$$\text{Ring} \rightarrow k = R \Rightarrow I = mR^2$$

$$\text{Disc} \rightarrow k = \frac{R}{\sqrt{2}} \Rightarrow I = \frac{mR^2}{2}$$

Maximum Fluctuation of energy :-

Variation

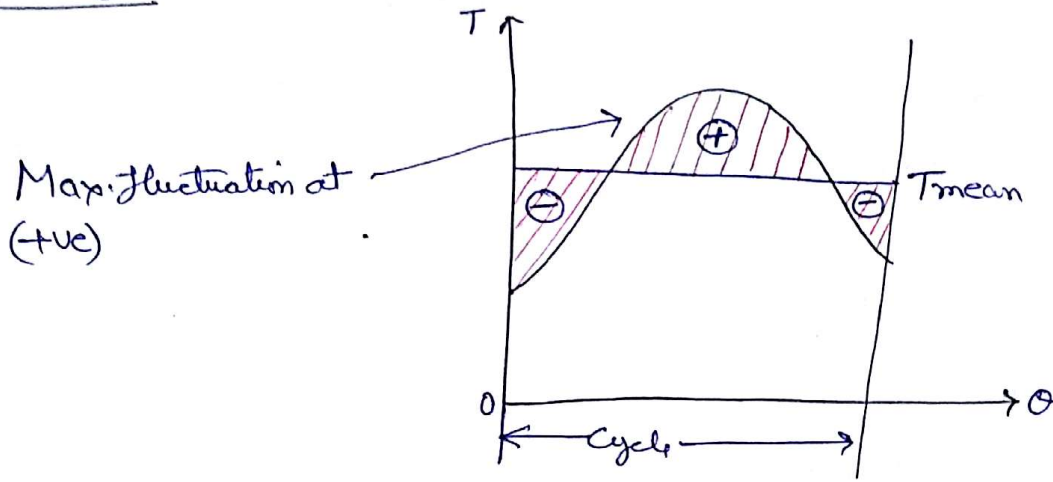
$$(\Delta E)_{\max} = E_{\max} - E_{\min}$$

$$= \frac{1}{2} I \omega_{\max}^2 - \frac{1}{2} I \omega_{\min}^2$$

$$= \frac{1}{2} I \left(\frac{\omega_{\max} + \omega_{\min}}{2} \right) \left(\frac{\omega_{\max} - \omega_{\min}}{\omega} \right) \cancel{2} \times \omega$$

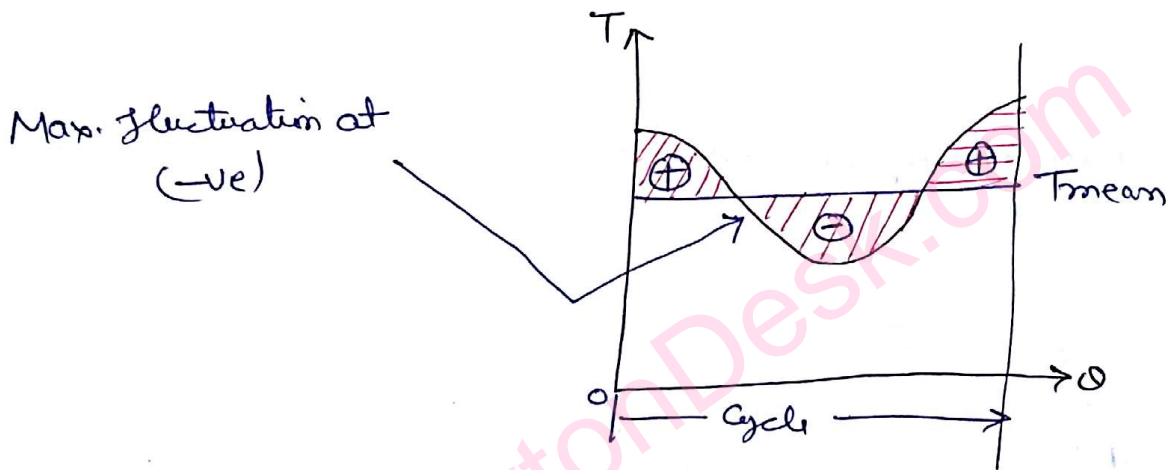
$$\boxed{(\Delta E)_{\max} = I \omega^2 C_s} \quad **$$

For. e.g



Max. fluctuation at (+ve)

②



Max. fluctuation at (-ve)

Period:

$\sin \theta \rightarrow 2\pi$ $\cos \theta \rightarrow 2\pi$ $\tan \theta \rightarrow \pi$	$\sin 3\theta = \frac{2\pi}{3}$ $\cos 3\theta = \frac{2\pi}{3}$ $\tan 7\theta = \frac{2\pi}{7}$
---	---

$$f(\theta) = 1000 + 200 \sin 3\theta + 300 \cos 5\theta + 700 \tan 7\theta$$

\downarrow Constant
 \downarrow $\frac{2\pi}{3}$
 \downarrow $\frac{2\pi}{5}$
 \downarrow $\frac{\pi}{7}$

$$\text{Period} = \frac{\text{L.C.M of Numerator}}{\text{HCF of Denominator}} = \frac{2\pi}{1} = 2\pi$$

Soln)

$$T = [10000 + 2000 \sin 2\theta - 1800 \cos 2\theta]$$

cycle $\rightarrow (0, \pi)$

$$C_s = \pm 0.25\% = 0.5\%$$

$$= 0.05$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} \text{ rad/s}$$

$$T_{\text{mean}} = \text{Const.}$$

$$W_{\text{cycle}} = \int_0^\pi T \cdot d\theta$$

$$= \int_0^\pi (10000 + 2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$$

$$W_{\text{cycle}} = 10000\pi \text{ Joules}$$

$$T_{\text{mean}} = \frac{10000 \cdot \pi}{\pi} = 10000 \text{ N-m} \quad T_{\text{mean}} = \frac{W_{\text{cycle}}}{\pi}$$

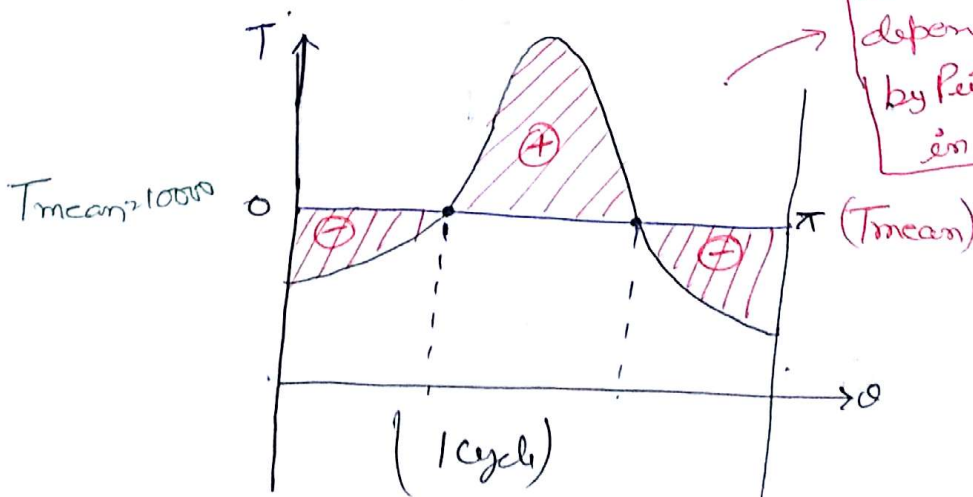
$$T_{\text{mean}} = 10000 \text{ N-m} \quad **$$

i)

$$\text{Power} \rightarrow P = T_{\text{mean}} \times \omega$$

$$P = 10000 \times \frac{2\pi \times 250}{60} \text{ Watt}$$

ii)



dirⁿ of Curve will depend on by putting Value of θ in T

Those Point, where T curve cut Tmean line,

At Those points:

$$T = T_{\text{mean}}$$

$$10000 + 2000 \sin^2 \theta - 1800 \cos 2\theta = 10000$$

$$\tan 2\theta = 0.9$$

$$2\theta = 41.8872^\circ, 221.9872^\circ, 401.9872^\circ \dots$$

$$\theta = 20.9936^\circ, 110.9936^\circ, 200.9936^\circ$$

$$\begin{aligned} \omega \sin \theta &= 2\pi \\ \omega \cos \theta &= 2\pi \\ \tan \theta &= 1 \end{aligned}$$

$$\Delta E_{\text{max}} = \int_{\theta_1}^{\theta_2} (T - T_{\text{mean}}) d\theta$$

$$= \int_{20.9936^\circ}^{110.9936^\circ} (2000 \sin 2\theta - 1800 \cos 2\theta) d\theta = I \omega^2 \epsilon_s$$

~~---~~

this I will return

$$\text{ii) } (T_{\text{mean}})_{\theta=45^\circ} = I \alpha$$

$$2000 = I \alpha$$

$$\alpha = \frac{2000}{I}$$

$$\boxed{\alpha = \frac{2000}{I} \text{ rad/s}^2}$$

Q.28)

$$T = 5000 + 1500 \sin 30 \left] \frac{2\pi}{3} \right.$$

$$I = 100 \text{ kg-m}^2$$

$$N = 300 \text{ rpm}$$

$$\omega = \frac{2\pi \times 300}{60} \text{ rad/s.}$$

$$C_s = ?$$

$$T_{\text{mean}} = 5000 + 600 \sin \theta \left] \frac{2\pi}{1} \right.$$

2π

(ii) (b)

These point where T curve cuts T_{mean} curve;

At those point

$$T = T_{\text{mean}}$$

$$5000 + \frac{1500}{5} \sin 30 = \frac{5000}{5} + \frac{600}{2} \sin \theta$$

$$5(3 \sin \theta - 4 \sin^3 \theta) = 2 \sin \theta$$

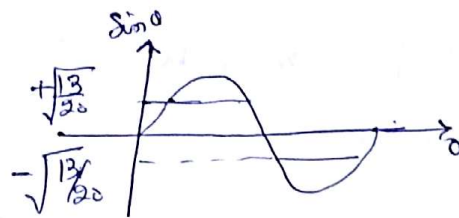
$$13 \sin \theta - 20 \sin^3 \theta = 0$$

$$\sin \theta (13 - 20 \sin^2 \theta) = 0$$

$$\sin \theta = 0 \Rightarrow 0, \pi, 2\pi$$

$$\sin^2 \theta = \frac{13}{20}$$

$$\sin \theta = \begin{cases} +\sqrt{\frac{13}{20}} \\ -\sqrt{\frac{13}{20}} \end{cases}$$



$$\rightarrow 53.7288^\circ$$

$$\rightarrow 126.2711^\circ$$

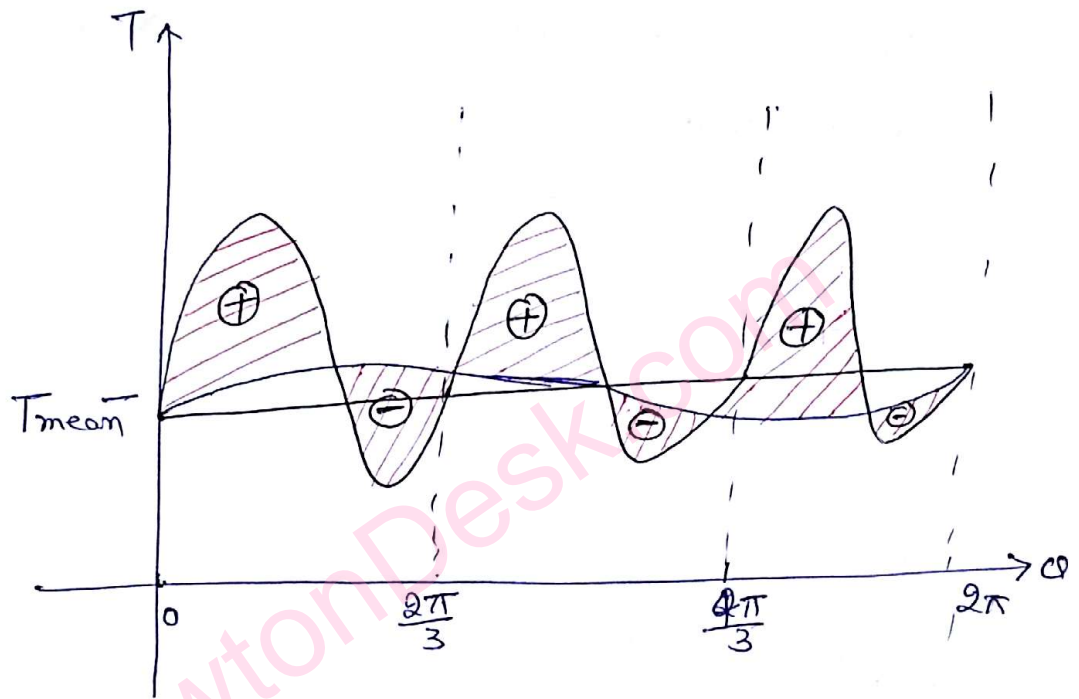
$$\rightarrow 233.7288^\circ$$

$$\rightarrow 306.2711^\circ$$

- 0° — ①
- 53.7288° — ②
- 126.2711° — ③
- 180° — ④
- 233.7288° — ⑤
- 306.2711° — ⑥
- 360° — ⑦

$$\int T - T_{\text{mean}}$$

$$= \int 1500 \sin 3\theta - 600 \sin \theta$$



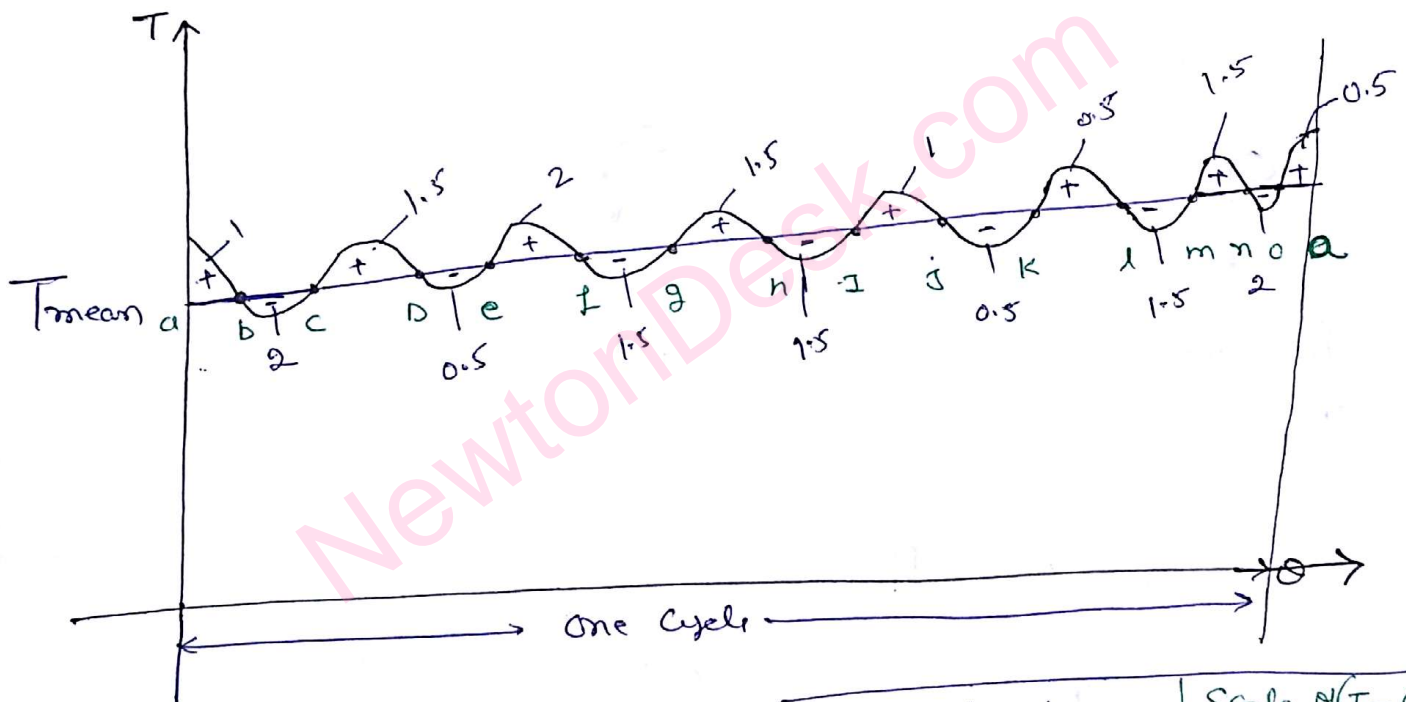
$$(DE)_{\text{max}} = \int_{233.72880}^{306.2711} (T - T_{\text{mean}}) d\theta = 9 = I\omega^2 C_s$$

$$C_s = 9$$

Requirement of Flywheel in multi cylinder Engines

- Multicylinder engine Concept, came into picture, to ↑ the Power as well as to increase the uniformity of the Power. So, that, the fluctuation in the engine performance can be decreased & the requirement of flywheel, can be suppressed up to certain limits.
- This is achieved by imposing a very nice concept known as firing orders.
- This concept is also playing very, very important role in the area of Balancing.

Let us assume no. of cylinder are more than 7-8



Let us assume

$$\begin{aligned} E_a &= E + 1 \\ E_b &= E + 1 \\ E_c &= E - 1 \\ E_d &= E + 0.5 \\ E_e &= E \\ E_f &= E + 2 \\ E_g &= E + 0.5 \end{aligned}$$

$$\begin{aligned} E_h &= E + 2 \\ E_i &= E + 0.5 \\ E_j &= E + 1.5 \\ E_k &= E + 1 \\ E_l &= E + 1.5 \\ E_m &= E \\ E_n &= E + 1.5 \\ E_o &= E - 0.5 \\ E_a &= E \end{aligned}$$

$$E_{max} = (E + 2) \quad (\text{at } f \& h)$$

$$E_{min} = (E - 1) \quad \text{at } c$$

$$\begin{aligned} (\Delta E)_{max} &= (E + 2) - (E - 1) \\ &= 3 \text{ mm}^2 \end{aligned}$$

$$= 3 \times 0.02 \text{ Joule}$$

then equate to Iω²c

Scale of (T - 0)
1 mm ² = 0.2 Joules

Requirement of Flywheel in Power Presses:

Power Press:

↓

used in Metal Forming process.

↓

Punching
Blanking
Shearing

Power Press Runs by motor

Flywheel at CAM Shaft

CAM



Follower
(Punch)

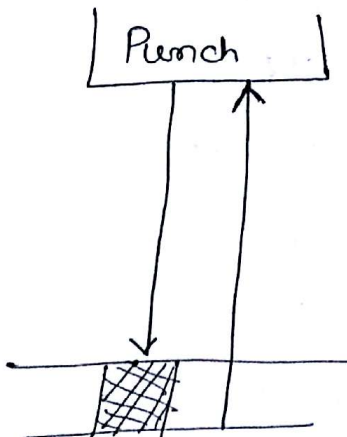
Flywheel at Crank Shaft

CRANK



Piston
(Punch)

One Cycle of Power Press:



→ Two Stroke are Covered

→ Time taken to Complete one cycle.

Cycle time

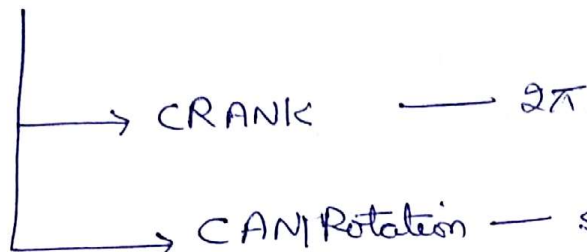
↓

Exact operation time $\ll \ll$ Cycle Time

↓

very less fraction

In One Cycle



Due to Dwell Stroke, in which Cam rotates but follower not.

Q.30) 720 holes/hr
 \Rightarrow 720 holes/3600sec
 \Rightarrow 1 hole/5 sec

$$r_c = 0.3m$$

$$\boxed{\text{Cycle time} = 5 \text{ Sec}}^*$$

$$1 \text{ hole} / \frac{1}{4} \text{ sec}$$

$$\text{exact punching time} = \frac{1}{4} \text{ sec.}$$

During Punching

$$100 \text{ rpm (Nmax)}$$

↓

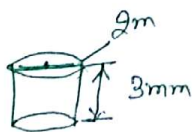
$$80 \text{ rpm (Nmin)}$$

$$N = \frac{100 + 80}{2} = 90 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \cdot 90}{60} \text{ rad/sec}$$

$$\omega_s = \frac{100 - 80}{90}^*$$

1 hole



$$A_{\text{sheared}} = \pi d T$$

$$= \pi \times 2 \times 3 = 6\pi \text{ m}^2$$

$$E_{\text{hole}} = 6\pi \times 20 = 120\pi \text{ Joules.}$$

Motors

No Punch time used

$$P_{\text{motors}} = \text{Energy required / sec}$$

$$= E_{\text{hole}} \times \text{No. of holes/sec} \rightarrow (\text{cycles time})$$

~~*** v. imp. !!~~

$$= 120\pi \times \frac{720}{3600}$$

$$= 24\pi \text{ Watt (J/s)}$$

Motors Installed :

$$P_{\text{punching}} = \left(\frac{1}{4} \text{ sec}\right)$$

$$E_{\text{available}} = 24\pi \times \frac{1}{4}$$
$$= 6\pi \text{ Joules}$$

$$E_{\text{hole}} = 120\pi \text{ Joules}$$

$$120\pi - 6\pi = 114\pi = I \omega^2 C_s$$

$$= mk^2$$

Variation in above Problem :

- ① Cycle time : 5 Sec
Exact Punching time = ?
Stroke length = 100 mm

$$\underline{200 \text{ mm} - \text{Stroke length}}$$

$$\begin{array}{l} 200 \text{ mm} \text{ --- } 5 \text{ Sec} \\ 3 \text{ mm} \text{ --- } \frac{5}{200} \times 3 \text{ Sec} \\ \downarrow \\ \text{thickness of sheet} \end{array}$$

thickness of sheet

- ② Cycle time = 5 Sec
Exact punching time = ?
 $\omega = ??$

Exact punching is done in 20° of crank rotation.

9 in one cycle \rightarrow Crank rotation = 2π in 5 Sec

$$\boxed{\omega = \frac{2\pi}{5} \text{ rad/sec}}$$

$$\begin{array}{l} 360^\circ \text{ --- } 5 \text{ Sec} \\ 20^\circ \text{ --- } \frac{5}{360} \times 20^\circ \\ \downarrow \\ \text{exact punching time} \end{array}$$

Q. 29

$$\text{Cycle time} = 2 \text{ sec}$$

$$P_{\text{motor}} = 1500 \text{ watt} = E_{\text{rot}} \times \frac{1}{2}$$

$$E_{\text{rot}} = 3000 \text{ Joules}$$

$$\omega = \frac{2\pi}{2} = \pi \text{ rad/s}$$

$$\text{exact Punching time} = \frac{1}{6} \text{ sec}$$

$$C_s = 20\% = 0.2$$

Punching ($\frac{1}{6}$ sec)

$$3000 - (1500 \times \frac{1}{6}) = I\omega^2 C_s$$

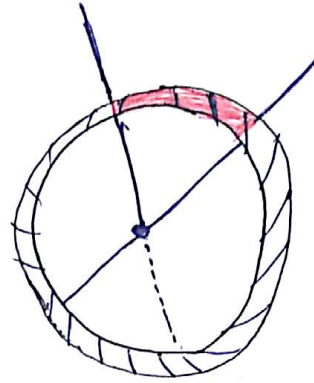
↓
~~299~~

→ Designing of Flywheel Rim :-

A → Rim cross section area

$$\text{Ring} \rightarrow I = mR^2$$

Max.

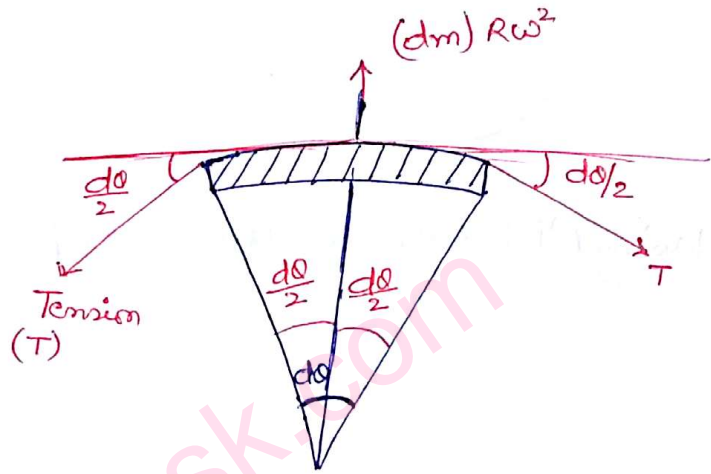


$$2T \sin \frac{d\theta}{2} = (dm) R \omega^2$$

$$2T \sin \frac{d\theta}{2} = (R d\theta A \cdot \rho) R \omega^2$$

$$\frac{T}{A} = \rho (R\omega)^2 = \rho v^2$$

↓
(Hoop's stress)



$$v = (R\omega) = \sqrt{\frac{\sigma}{\rho}}$$

→ Hoop's stress

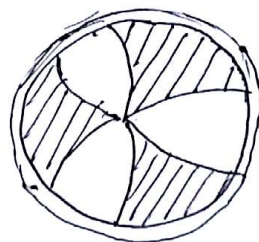
$$\sigma \leq \sigma_b$$

where σ_b is bearing limit for Hoop's stress

$$v_{max} = (R\omega)_{max} = \sqrt{\frac{\sigma_b}{\rho}}$$

At Medium speed :-

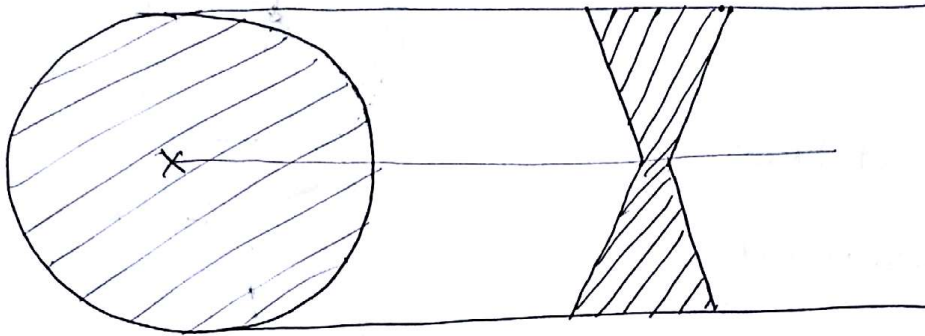
Flywheel with spokes.



At high speed :- Disc shaped flywheel

Best flywheel due to

$$\frac{1}{2} I \omega^2$$



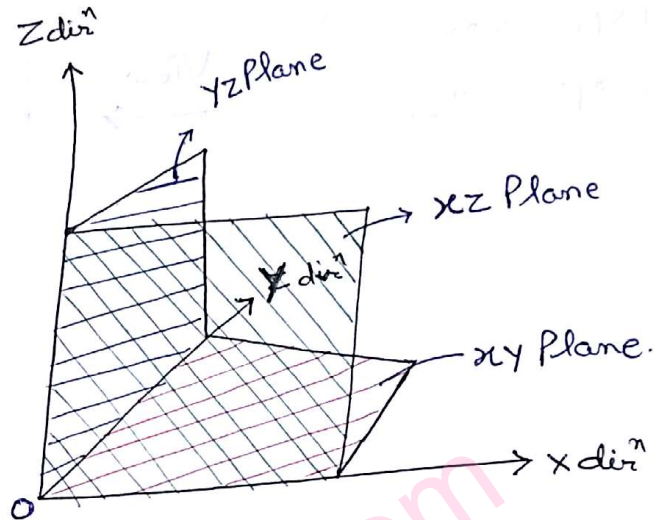
Having highest energy storing capacity $\rightarrow \frac{1}{2} I \omega^2$

Ring

$$I < mR^2$$
$$> \frac{mR^2}{2}$$

Gyroscope

3-D Space Co-ordinate System :-



Concept of Angular velocity ($\vec{\omega}$) :-

Rate of Change of Angular displacement.

$\vec{\omega} \rightarrow$ Vector Quantity

Magnitude

(Rate of change of Ang. Displacement)

Direction \rightarrow (Dirⁿ of $\vec{\omega}$ is Also known as Axis of Spin)



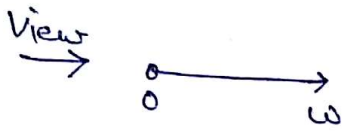
By Right hand thumb Rule.

Right hand thumb rule :-

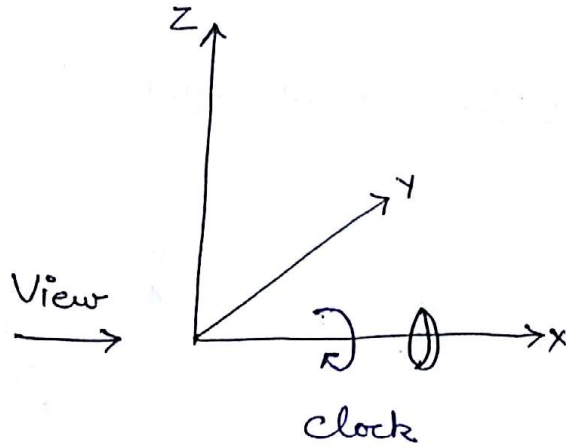
- i) Place the Right hand in such a way, such that thumb is \perp to the fingers
- ii) Rotate the fingers in the sense of Rotation, then the direction of thumb will be the dirⁿ of Angular velocity.

Examples:

1)



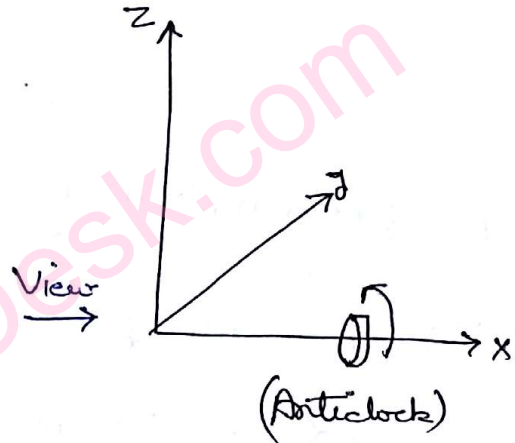
Axis of Spin $\rightarrow OX$
Plane of Spin $\rightarrow YZ$



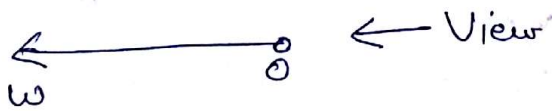
2)



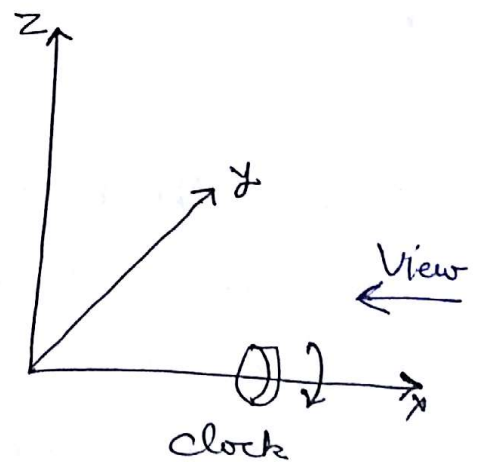
Axis of Spin $- OX$
Plane of Spin YZ



3)



Axis of Spin $- OX$
Plane of Spin $- YZ$



Angular Acceleration α ($\vec{\alpha}$)

\Downarrow
Rate of Change of Angular velocity.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

$\vec{\omega} \rightarrow$ Vector Quantity $\begin{cases} \text{Mag} \\ \text{dir}^n \end{cases}$

α may exist :- bec of

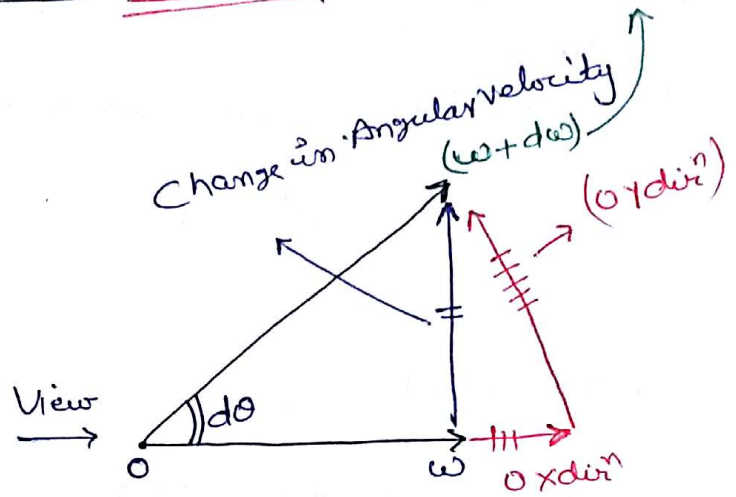
- \rightarrow Bec of the Change in Magnitude of Angular velocity.
 - \rightarrow Bec of the Change in direction of Angular velocity. \rightarrow (Precession)
- \rightarrow Let us Consider a Case, in which Magnitude of $\vec{\omega}$ & the dirⁿ of $\vec{\omega}$ both are Changing.



Angular Velocity Diagram :

e.g. Table Fan

NewtonDesk.com After time dt



Angular acceleration due to change in Magnitude of Angular Velocity :-

$$= \frac{(\omega + d\omega) \cos d\theta - \omega}{dt}$$

$$= \frac{(\omega + d\omega) - \omega}{dt}$$

Ang. Acceleration due to change in Magnitude of Ang. velocity ($\vec{\omega}$) = ~~$\frac{d\omega}{dt}$~~ $\frac{d\omega}{dt}$ OX direction

Angular Acceleration due to change in dirⁿ of Angular Velocity

$$= \frac{(\omega + d\omega) \sin d\theta - 0}{dt}$$

$$= \frac{(\omega + d\omega) \cdot d\theta}{dt}$$

$$= \frac{\omega \cdot d\theta + d\omega \cdot d\theta}{dt}$$

$$= \omega \frac{d\theta}{dt} \cdot (\omega) \text{ dir}^n$$

Angular Velocity or Precession (ω_p)

$$\boxed{= \omega \cdot \omega_p}$$

OY dirⁿ

Total Average Acceleration:

$$\vec{\alpha} = \left(\frac{d\omega}{dt}\right) \hat{i} + \omega \left(\frac{d\theta}{dt}\right) \hat{j}$$

$$\Rightarrow \vec{\alpha} = \left(\frac{d\omega}{dt}\right) \hat{i} + \underbrace{(\omega \cdot \omega_p)}_{\substack{\downarrow \\ \text{(gyroscopic acceleration)}}} \hat{j}$$

If the magnitude of Angular velocity is not changing, only the dirⁿ is changing, then;

$$\frac{d\omega}{dt} = 0$$

$$\alpha = \omega \cdot \frac{d\theta}{dt}$$

$$\alpha = \omega \cdot \omega_p \quad \text{in OY direction}$$

To have the precession, i.e. to change the dirⁿ of $\vec{\omega}$ in order to provide Angular acceleration $(\alpha) = \omega \cdot \omega_p$ in OY dirⁿ

→ A torque is required; and that requirement of the torque is known as Active Gyroscopic Couple (C)

$$\text{Active gyroscopic Couple (C)} = I \alpha$$

$$C = I \omega \cdot \omega_p \quad \text{** OY direction}$$

Torque required to will be given to the system by external Agency

e.g. to turn Bike

↓
We made to turn Handle

Therefore, the Similar Couple will be experienced by the external agency in Reverse direction, i.e known as **Reactive Gyroscopic Couple**.

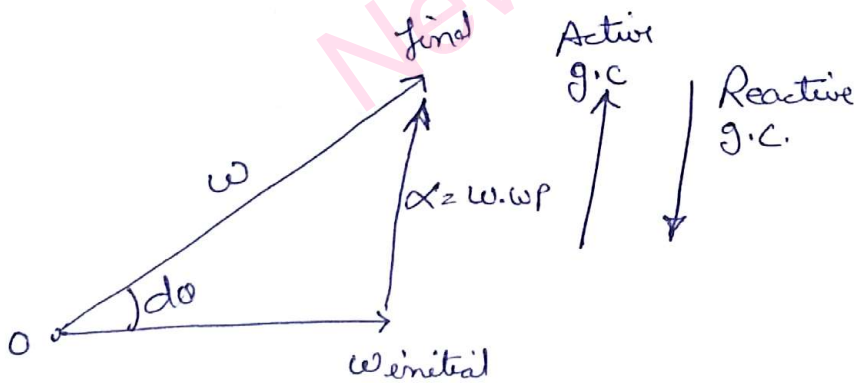
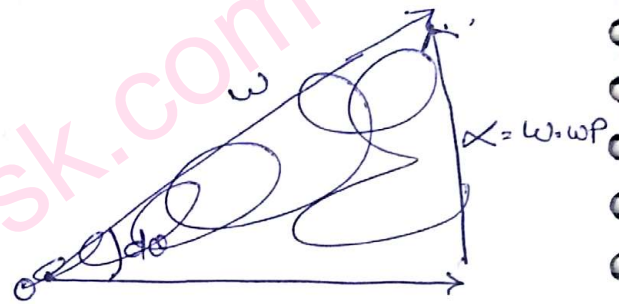
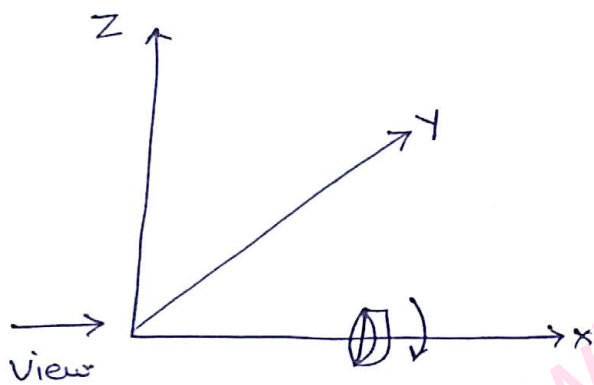
$$C = I \omega \omega_p$$

Thus, this Reactive gyroscopic Couple is experienced by the system because that external agency is the part of the system.

motor-table fan

After Effect of Gyroscopic Actions on the System :-

Reactive gyroscopic Couple.



Ox — Axis of Spin

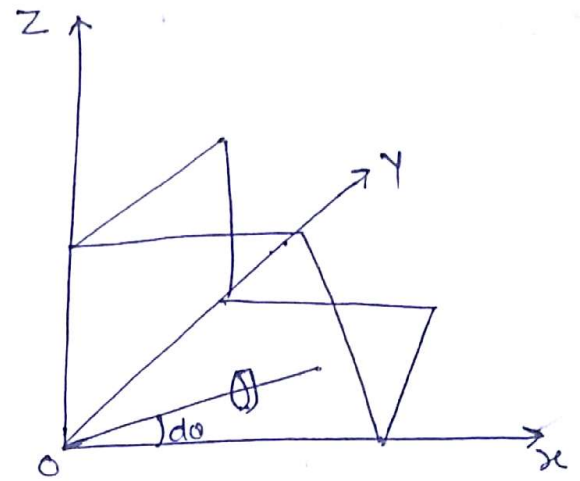
yz — Plane of Spin

Oz — Axis of Precession

xy — Plane of Precession

Oy — Axis of Gyroscopic Couple

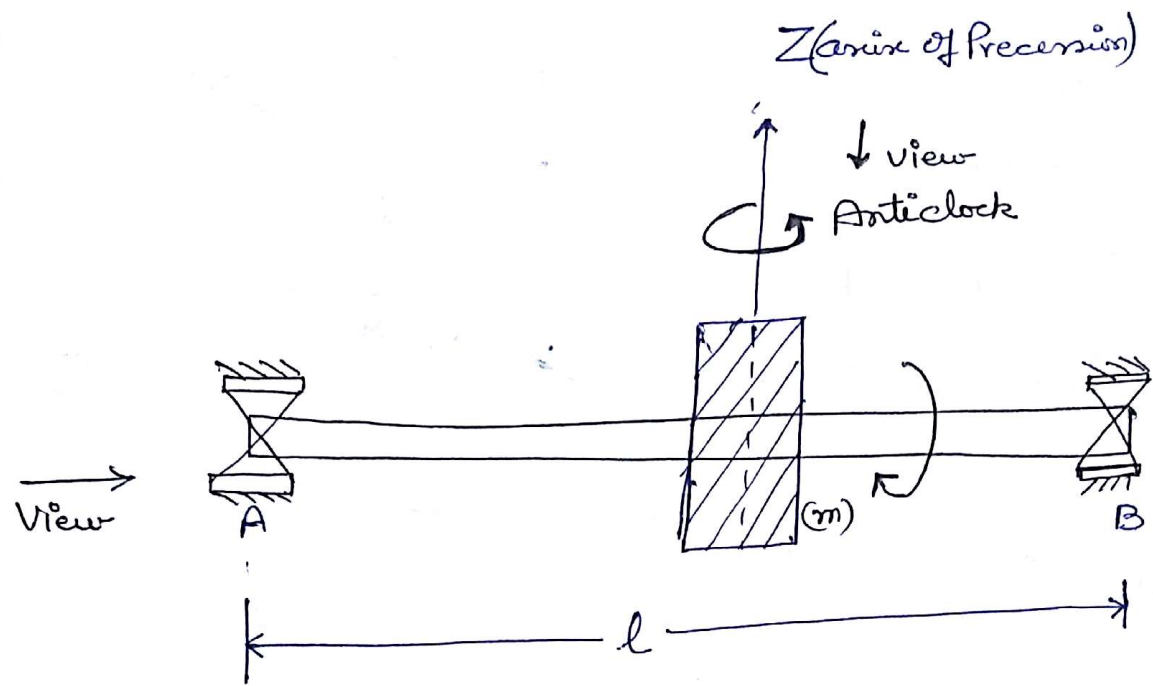
xz — Plane of Gyroscopic Couple



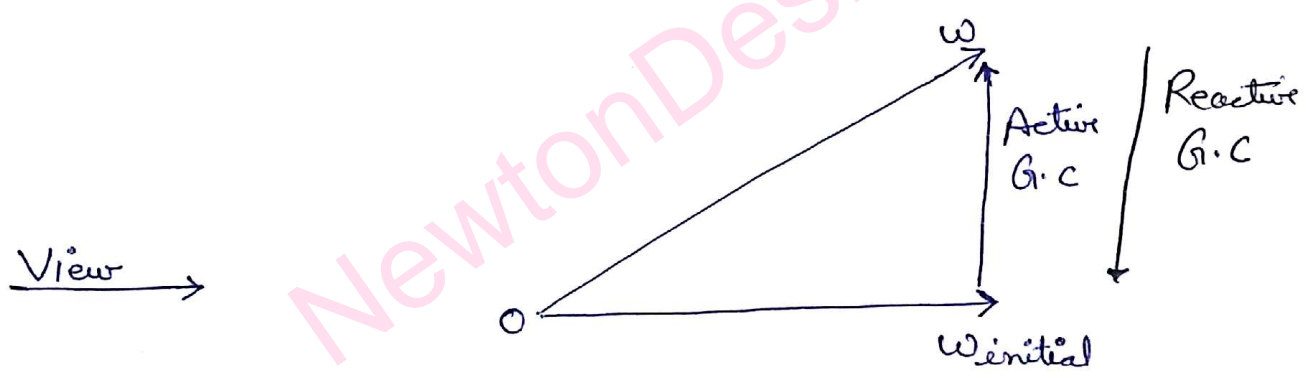
Applications :-

NewtonDesk.com

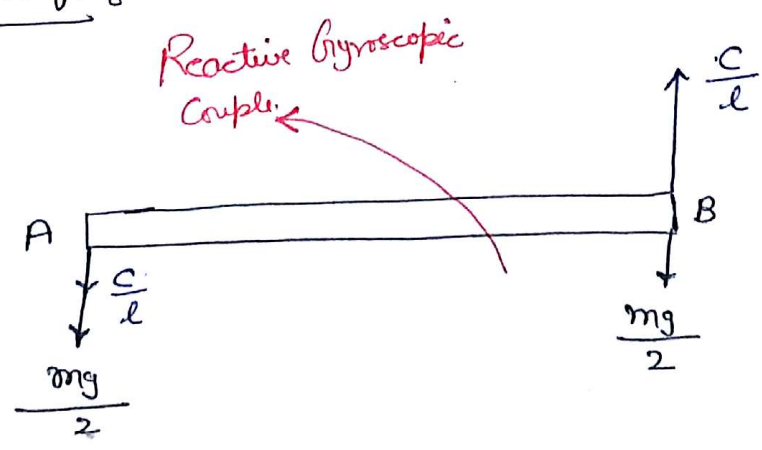
→ Reaction On bearings Under Gyroscopic Effect :-



$$C = I \omega \cdot \omega_p$$



Forces of Shaft :-



$$F_A = \left(\frac{C}{l} + \frac{mg}{2} \right) \text{ (downward)}$$

$$F_B = \left(\frac{C}{l} - \frac{mg}{2} \right) \text{ (upward)}$$

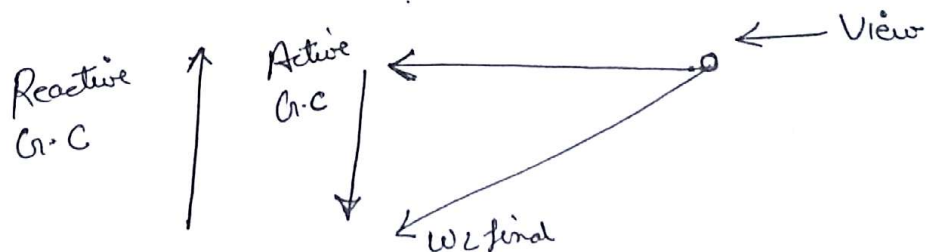
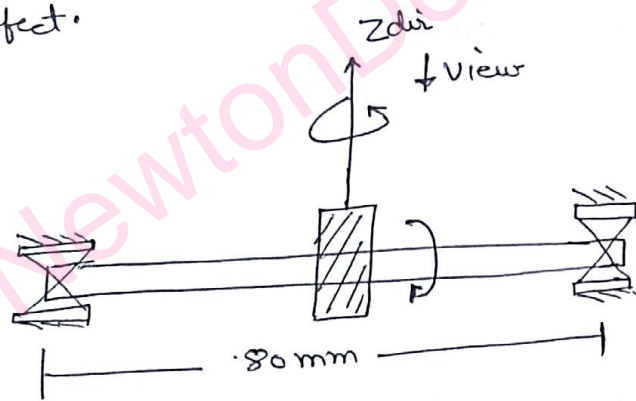
Reaction on bearing

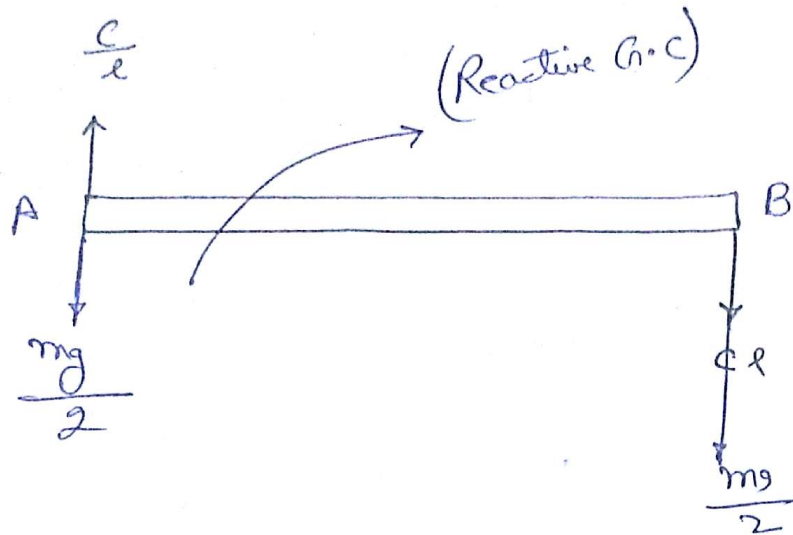
$$R_A = \left(\frac{C}{l} + \frac{mg}{2} \right) \text{ (upward)}$$

$$R_B = \left(\frac{C}{l} - \frac{mg}{2} \right) \text{ (downward)}$$

Qn) A disc with Radius of gyration 60 mm & mass 4 kg is mounted centrally on horizontal axis of 80 mm length b/w the bearings. It spins about the axis at 800 rpm (clockwise) when viewed from the right hand side bearing. The axis precesses about a vertical axis at 50 rpm in anticlockwise direction when viewed from above. Determine the Resultant R_m in each bearing due to the mass & gyroscopic effect.

Soln)





$$F_B = \left(\frac{c}{l} + \frac{mg}{2} \right) \text{ (downward)}$$

$$F_A = \left(\frac{c}{l} - \frac{mg}{2} \right) \text{ upward}$$

Reactions on Bearings :

$$R_B = \left(\frac{c}{l} + \frac{mg}{2} \right) \text{ upward}$$

$$R_A = \left(\frac{c}{l} - \frac{mg}{2} \right) \text{ downward.}$$

Given that;

$$m = 4 \text{ kg}$$

$$k = 0.060 \text{ m}$$

$$I = mk^2 = 4 \times (0.060)^2 = 0.0144$$

$$N = 800 \text{ rpm}$$

$$\omega = \frac{2\pi \cdot 800}{60} = \dots \text{ rad/s}$$

$$N_p = 50 \text{ rpm}$$

$$\omega_p = \frac{2\pi \cdot 50}{60}$$

$$C = I \omega \cdot \omega_P = ?$$

$$l = 0.080 \text{ m}$$

$$\frac{C}{l} = ?$$

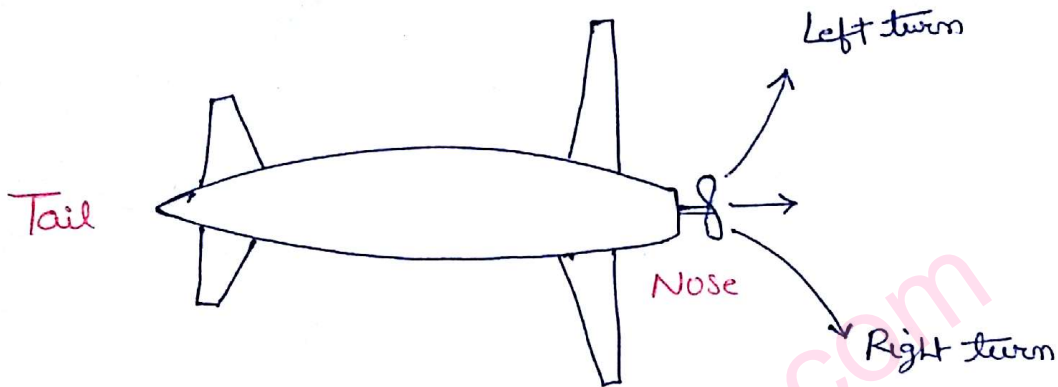
NewtonDesk.com

Gyroscopic Effect on Aircraft turning :-

(left or right)

→ Front end of Aircraft is known as Nose.

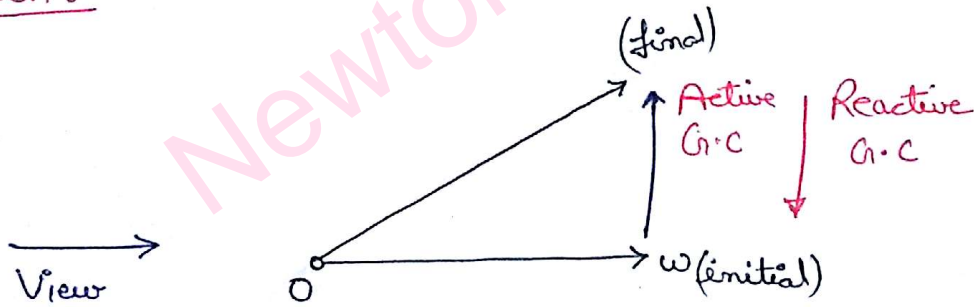
→ Rear end of Aircraft is known as Tail.



→ View from Tail (Rear end) :-

Engine Rotate Clockwise

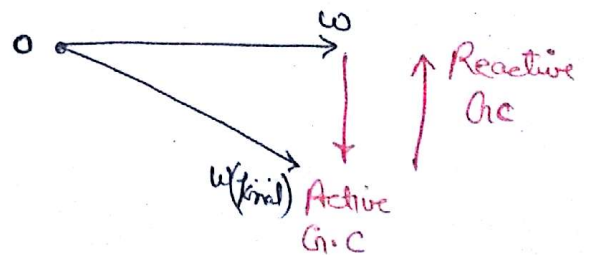
1) Left turn :-



Effect :- Nose will go up & tail will go down.

2) Right turn :-

Effect :- Nose will go down & Tail will go up.

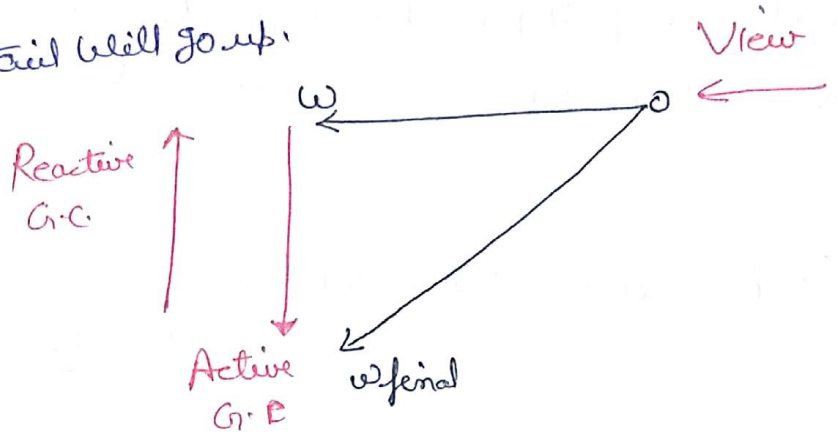


View from Nose (Front end):

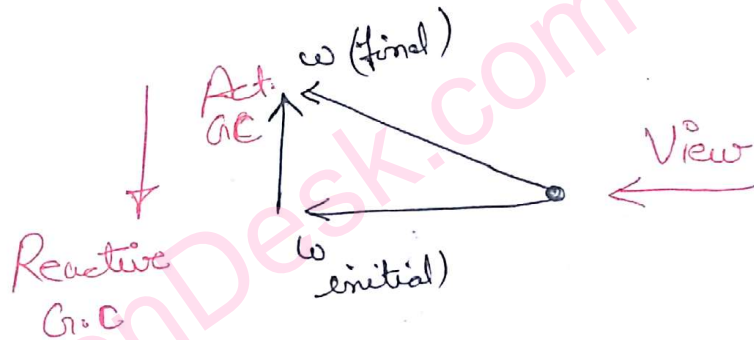
Engine Rotates clockwise

1) Left turn:

effect = Nose will go down & Tail will go up.



2) Right turn:



Effect :- Nose will go up & Tail will go down.

Gyroscopic effect on turning of ship:

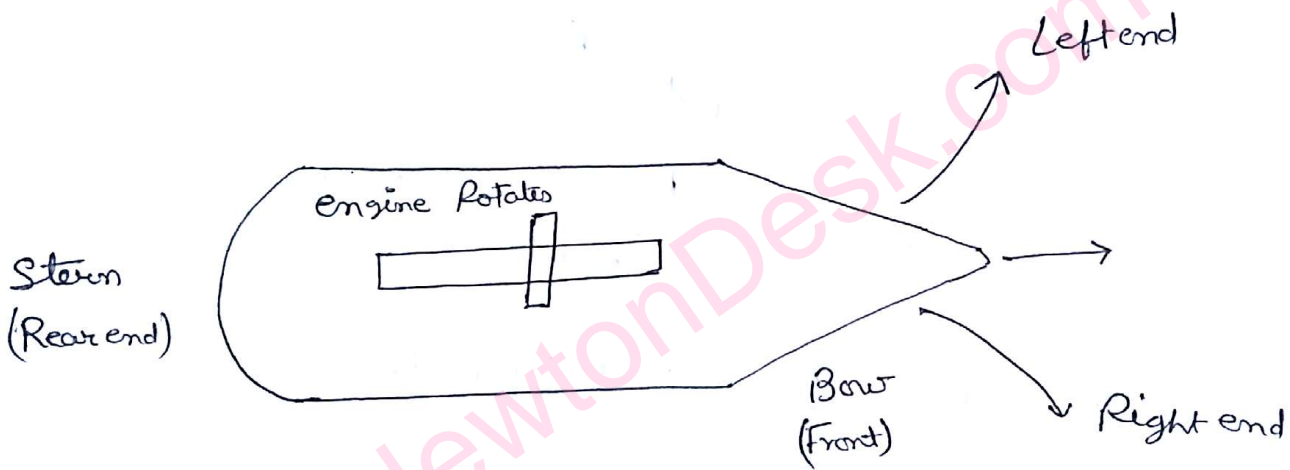
Nomenclature:-

Front end of ship is known as bow.

Rear end of ship is known as stern.

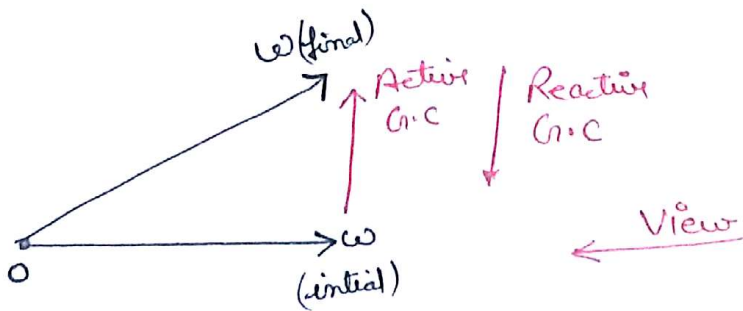
Looking from Stern side:- The Right hand side of ship portion is Starboard.

Left hand side of ship portion is Port.



1) View from Bow (Front): engine Rotates Anti clockwise

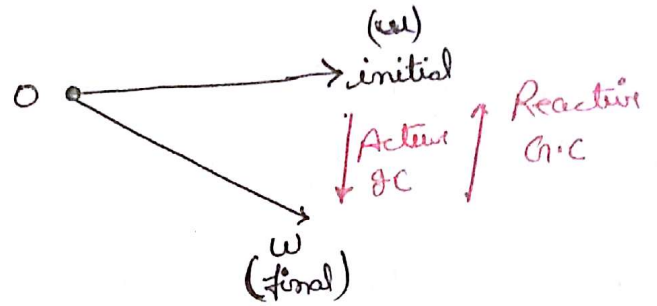
1) Left turn:



Effect \rightarrow Bow will move up & Stern will go/move down.

2) Right turn:

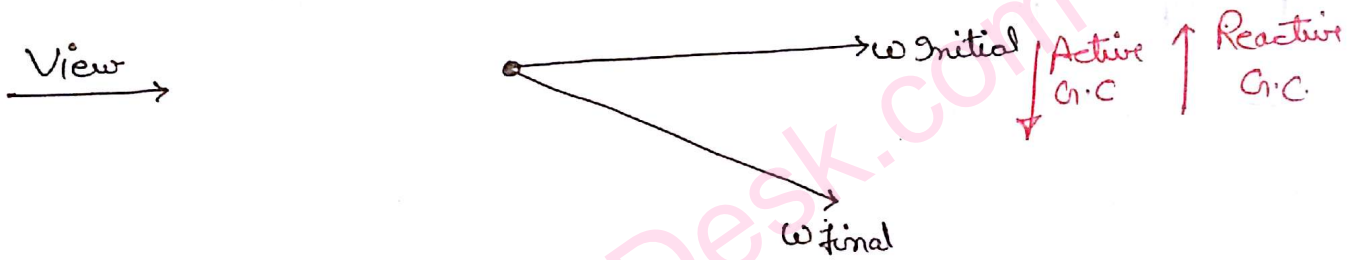
Effect:- Bows will move down & Stern will moves up.



Gyroscopic Effect on ship During Pitching: Engine Rotates Clockwise.

(View from Stern)

1) Both moves down (Stern moves up) during Pitching:

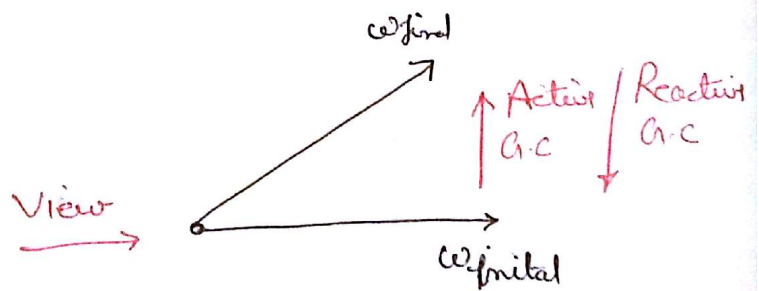


Effect:- Ship will try to turn towards Port side

2) Bow moves up (Stern moves down) during Pitching:

Effect:

Ship will try to turn towards Starboard side.



Gyroscopic effect on Rolling of the Ship :- NewtonDesk.com

During Rolling, of the ship, the direction of angular velocity (ω) of rotor of ship is not changing.

There is no Precession at all.

Hence gyroscopic Couple ($I\omega \cdot \omega_p$) is zero, bcs ω_p is zero.

∴ Ship will not experience any gyroscopic effect during Rolling

→ Gyroscopic effect on 4 Wheeler during turning :-

$I_E \rightarrow$ M.I of Engine Part (Rotating)

$\omega_E \rightarrow$ Speed of Engine

$I_w \rightarrow$ M.I of wheel

$\omega_w \rightarrow$ Speed of wheel

$\omega_p \rightarrow$ Speed of Precession

Gyroscopic effect on engine :-

$$C_E = I_E \omega_E \omega_P$$

Gyroscopic effect on wheel :-

$$C_w = 4 I_w \omega_w \omega_P$$

Total Gyroscopic effect on 4-wheeler :-

$$C = C_E + C_W$$

$$C = I_E \omega_E \omega_P + (I_W \omega_W \omega_P) 4$$

$$C = [I_E + 4 I_W \omega_W] \omega_P$$

During Turning : Gyroscopic effect

Force on outer wheel = $\frac{C}{2l}$ downward
(each) $l \rightarrow$ axle length b/w inner & outer wheel

Force on inner wheel = $\frac{C}{2l}$ upward.
(each)

Problem

Q₇) A high speed ship is driven by turbine motor having $m_o I$, 20 Kg-m^2 & is running at 3000 rpm in clockwise dirⁿ, when viewed from the Bow. The ship is speeding at 72 km/hr & taking a right turn round a curve of 600 m radius.

Determine the G.C applied to ship & its effects.

$$I = 20 \text{ Kg-m}^2$$

$$N = 3000$$

$$\omega = \frac{2\pi \times 3000}{60}$$

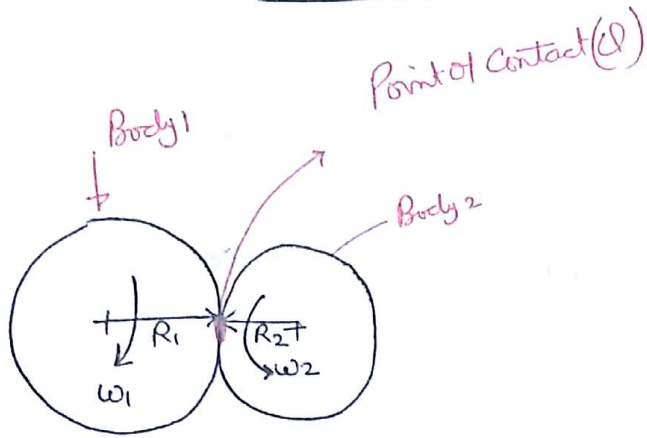
$$\omega = 314.16 \text{ rad/sec}$$

$$\omega_P = \frac{72 \times \frac{5}{18}}{600} = 0.10333 \text{ rad/sec}$$

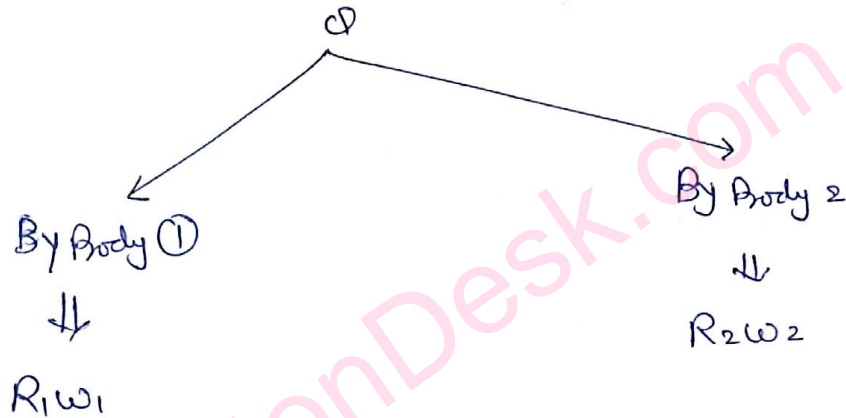
$$C = I \cdot \omega \cdot \omega_P$$

$$C = \underline{209.23}$$

Gears



Point of Contact



$$\boxed{R_1 \omega_1 = R_2 \omega_2}$$

Pure Rolling
(No slip)

Pure rolling :

$$\rightarrow \frac{\omega_1}{\omega_2} = \frac{R_1}{R_2}$$

→ Static friction is there.

$$\boxed{0 \leq f_s \leq \mu N}$$

All these Drives in which slips is possible

≠
negative drives

for example:

- Belt Drives
- Chain Drives
- Rope Drive

In Case of Slip

→ $\frac{\omega_1}{\omega_2} \neq \text{Constant}$

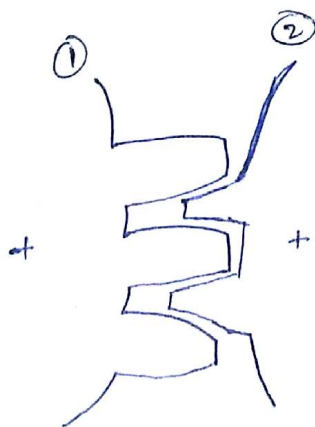
→ Kinetic friction.
(Some power loss)

In Some Cases; Positive drives → Gear Drive

Requires accuracy
[20-30% of
Power Transmission]

In Power Transmission, a very high level of Accuracy is demanded for the Velocity Ratio to be Constant.

$\frac{\omega_1}{\omega_2} = \text{Constant}$



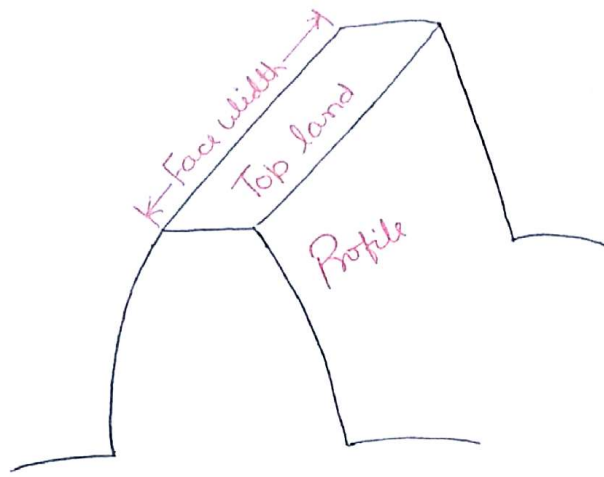
Slip is impossible



Positive drives

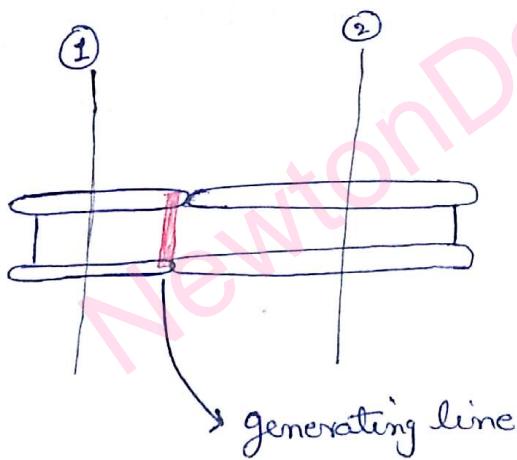


Gear Drive

Gear in 3DClassification of Gears :-

A) According to the Axes of shaft Connected :-

i) Both Axes are Parallel :-



Pure Rolling motion can be Transmitted b/w two cylindrical Surface in Contact.

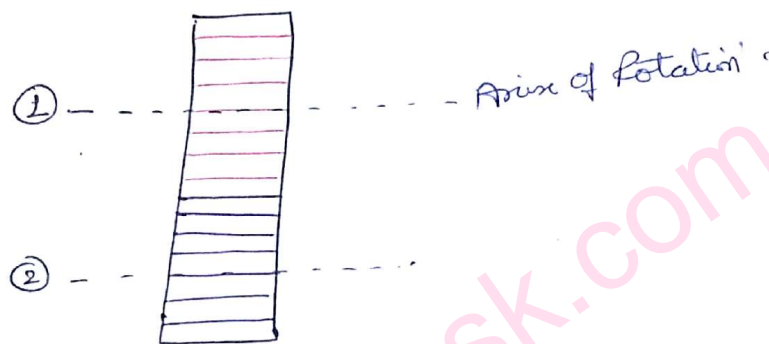
Spur Gear:

Teeth are straight and \parallel to axis of Rotation.

Use: — 99% failed & only 1% is used. In very low Power Transmissions at very Low Speed

Reason — Instantaneous engagement & Disengagement due to which Impact stresses on the profile.

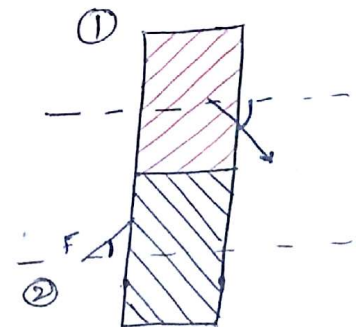
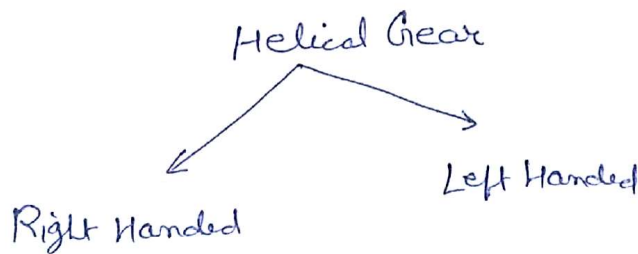
But having No Axial thrust



Helical Gear: 98% used Axial thrust is there

Modification of Spur Gear. "Teeth are straight but inclined to axis of Rotation."

There are two Type of Helical Gear:-



→ Always Opposite hand helical gear must come in Contact.

→ Gradual engagement

→ No impact stresses

→ Axial thrust at very high speed then it failed

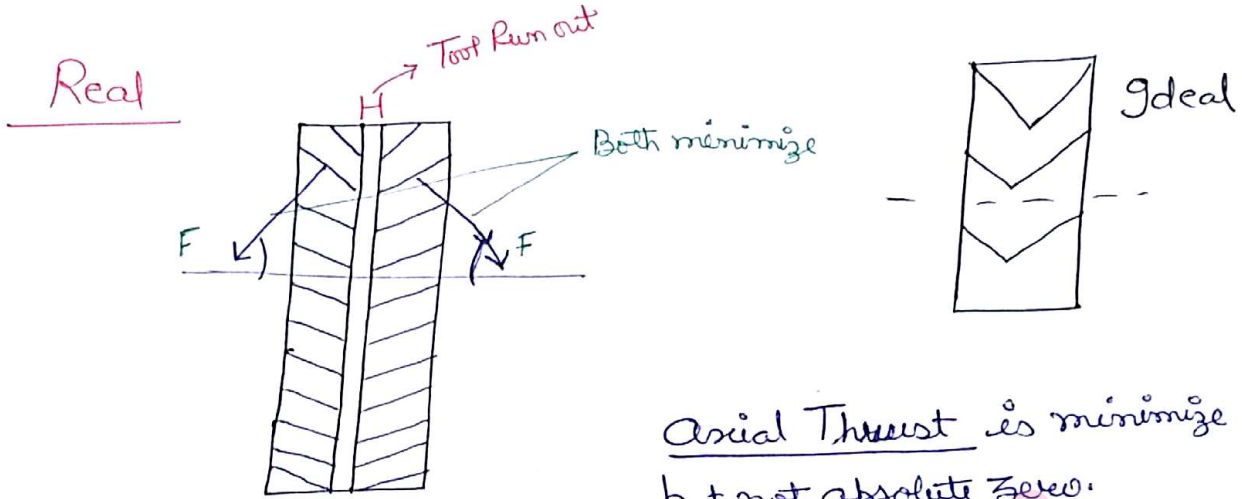
Note:-

due to high torque Axial thrust is there as axial thrust is there, overcome by bearing but at high Torque axial thrust is high and bearing where failed

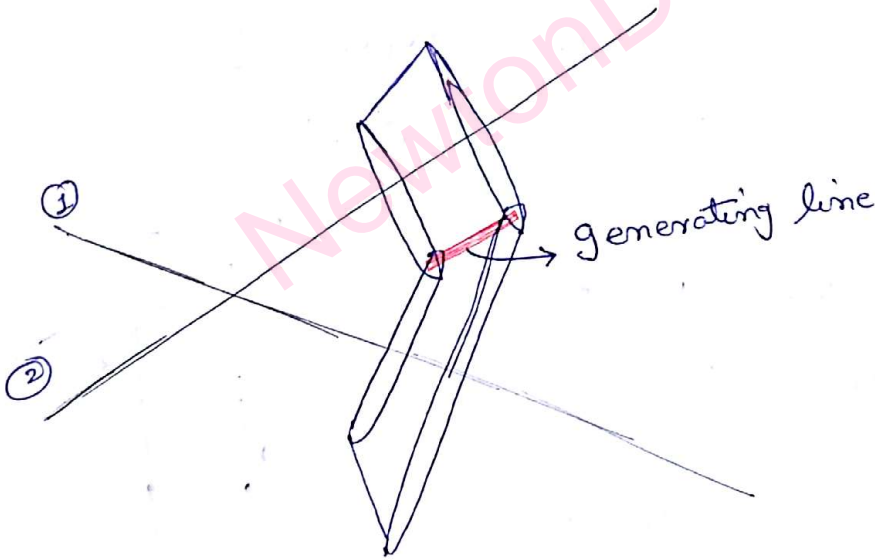
Double Helical Gear: For Very high Torque

used to minimize the Axial thrust.

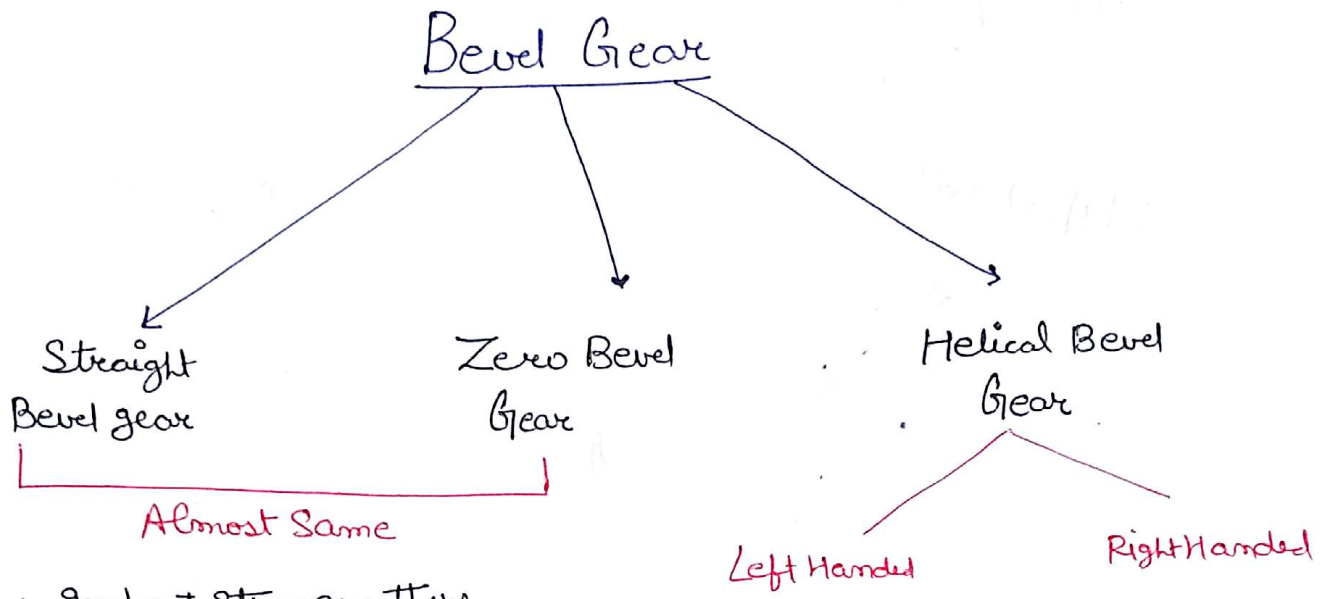
Given by Herringbone & Named Herringbone Gear.



ii) Axes are Non Parallel but Intersecting :-

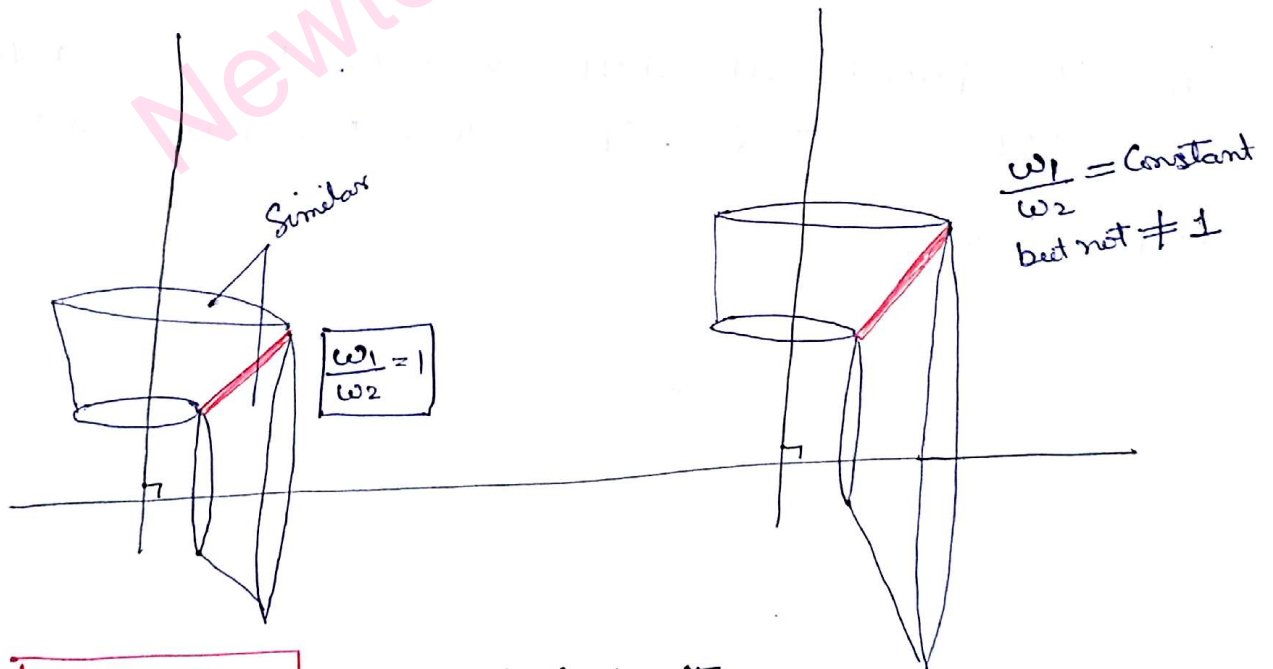
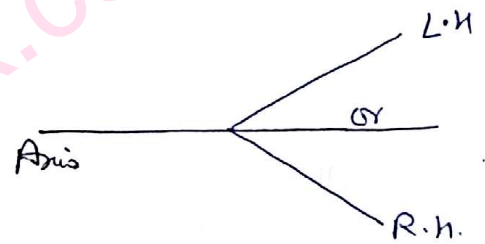


→ Pure Rolling motion, can be transmitted b/w two conical surfaces in contact.



→ Impact stresses there
1% used.

PLAN (TOP VIEW)



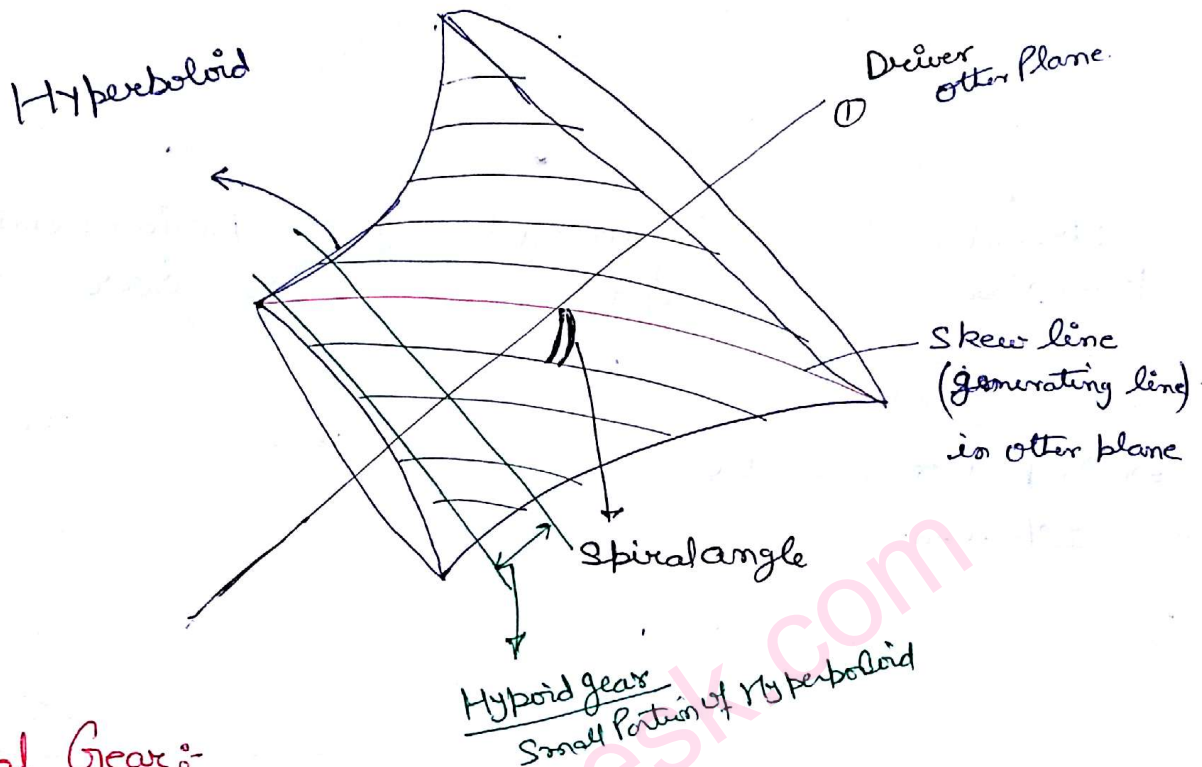
Mitre Gear → Use to Couple the Governor with the engine

iii) Axes are Neither Parallel nor Intersecting ©NewtonDesk.com

→ Pure Rolling is not Possible.

→ Rolling is possible.

↳ (Rotation + Partial Sliding)



Spiral Gear :-

⇓
Skew Bevel Gear

When the space b/w the shafts is very less then some Portion of Hyperboloid is used to form spiral gear. which are called Hypoid Gear.

Worm & Worm wheels :

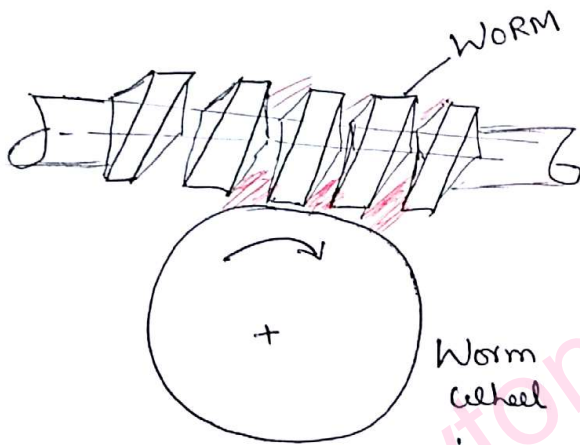
- Very less diameter
- Very high Spiral Angle.

- Very high diameters
- Very less spiral angle.

Worm is driver

Used in high Speed Reduction Ratio's

W	WW
10	1
30	1
300	1
1000	1
1250	1



Note

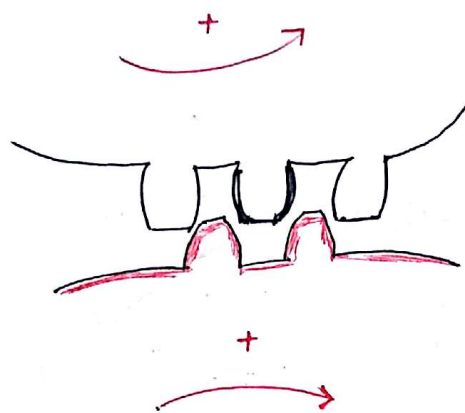
- ① WORM — Rotates
Worm wheel bcz
Worm overcome starting torque of worm wheel so that it rotates & slides

- ② but in worm wheel can't rotate worm so it happen called self locking
↓
cause of self locking

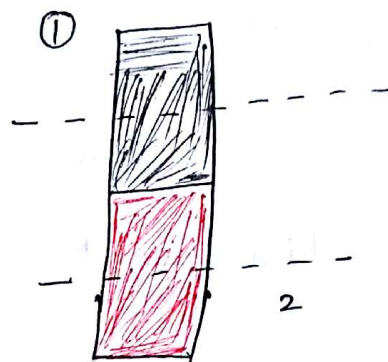
- ③ → Screw Jack

B) According to the type of Gearing :- NewtonDesk.com

External Gearing :



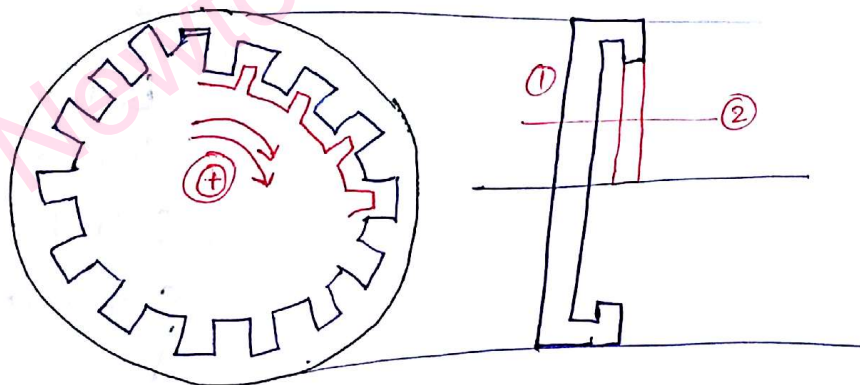
Top View



Driver → Smaller one (Pinion)

Bigger — Gear
Smaller — Pinion

Internal Gearing



Bigger → Annular (Ring)
Smaller → Pinion

Note :

→ If More than one gears are mounted on Same Shaft.

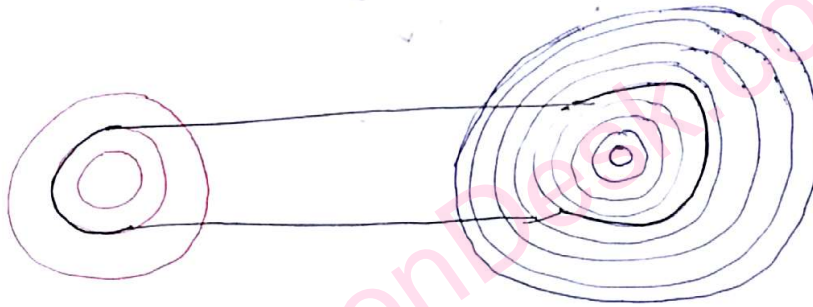
- Compound Gears
- Speed Same

→ Generally in power Transmissions smaller bodies are made as Drivers.

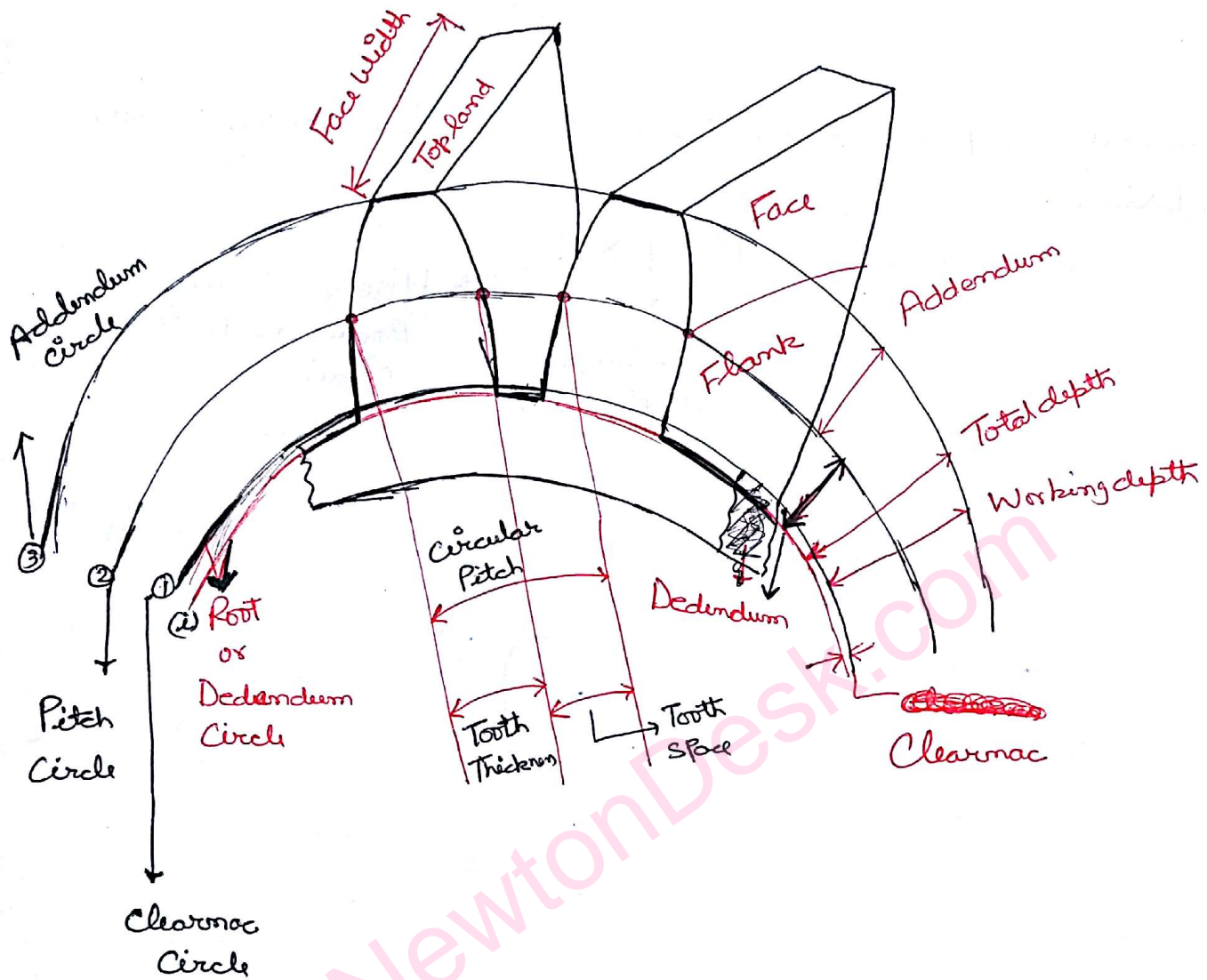
$$P = T \times \omega$$

↓
less Torque
is Required

→ High for Smaller
Bodies due to less
Radius.

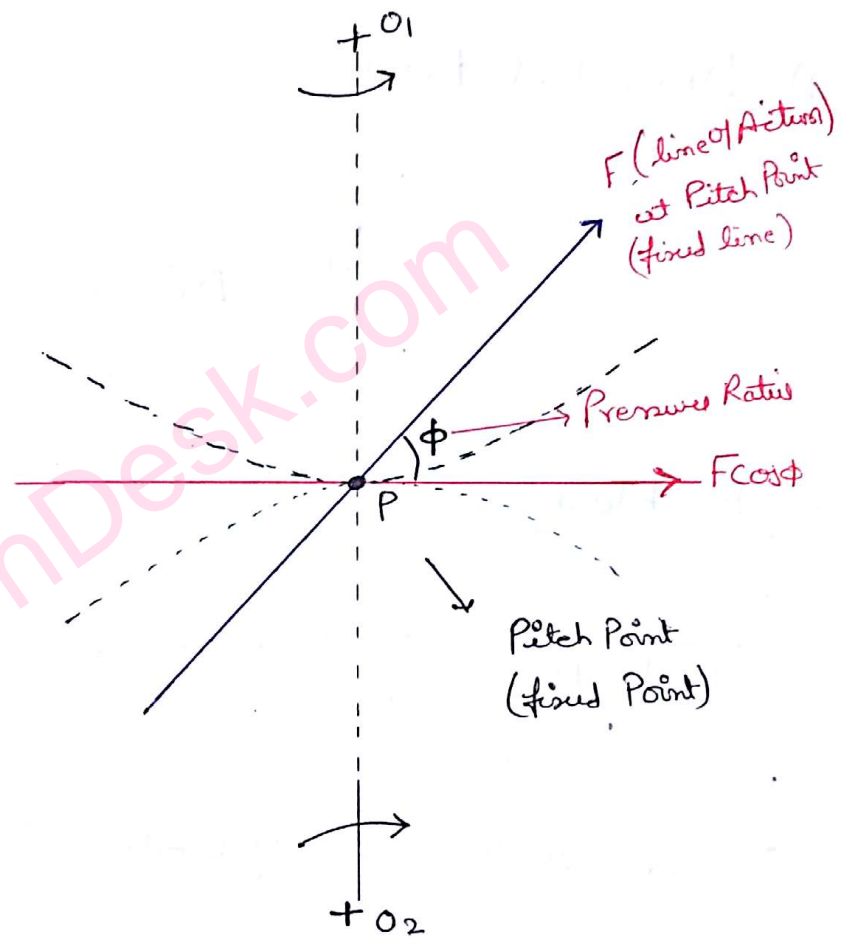


General Gear Terminology :-



Pitch Circle :- Most imp. circle \rightarrow where pure Rolling occurs. NewtonDesk.com

" It is the imaginary circle in the gears, where the pure rolling motion is observed, where the mating gears are transmitting Power. Being an imaginary circle, it can't be the physical characteristic of gear, but being the most important circle, it is one of the biggest specification of the Gears. The size of any gear, is specified by the diameter of pitch circle."



2) Circular Pitch (P_c) :-

Pitch circle diameter = D

No of teeth = T

$$P_c = \frac{\pi D}{T}$$

For two Mating Bodies :-

$$P_{c1} = P_{c2}$$

$$\frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}$$

$$\frac{D_1}{T_1} = \frac{D_2}{T_2}$$

3) Module (m)

Fixed Parameter is Design

$$m = \frac{D(\text{mm})}{T}$$

For two Mating Bodies

$$m_1 = m_2$$

4) Diametral Pitch :

$$P_d = \frac{T}{D(\text{inches})}$$

Ans.

$$P_c \cdot P_d = \frac{\pi D}{T} \times \frac{T}{D}$$
$$P_c \cdot P_d = \pi$$

5) Backlash :

Tooth Space - Tooth thickness of mating Gear = Backlash

To Prevent Jamming of ~~teeth~~ teeth ~~due to~~ due to thermal expansion,

Both gear & pinion are have to provide Backlash

6) Pressure Angle (ϕ)

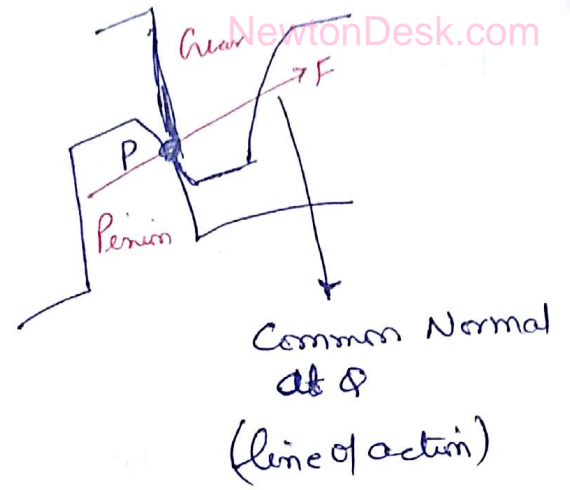
$\phi \rightarrow$ Pressure angle

\Downarrow

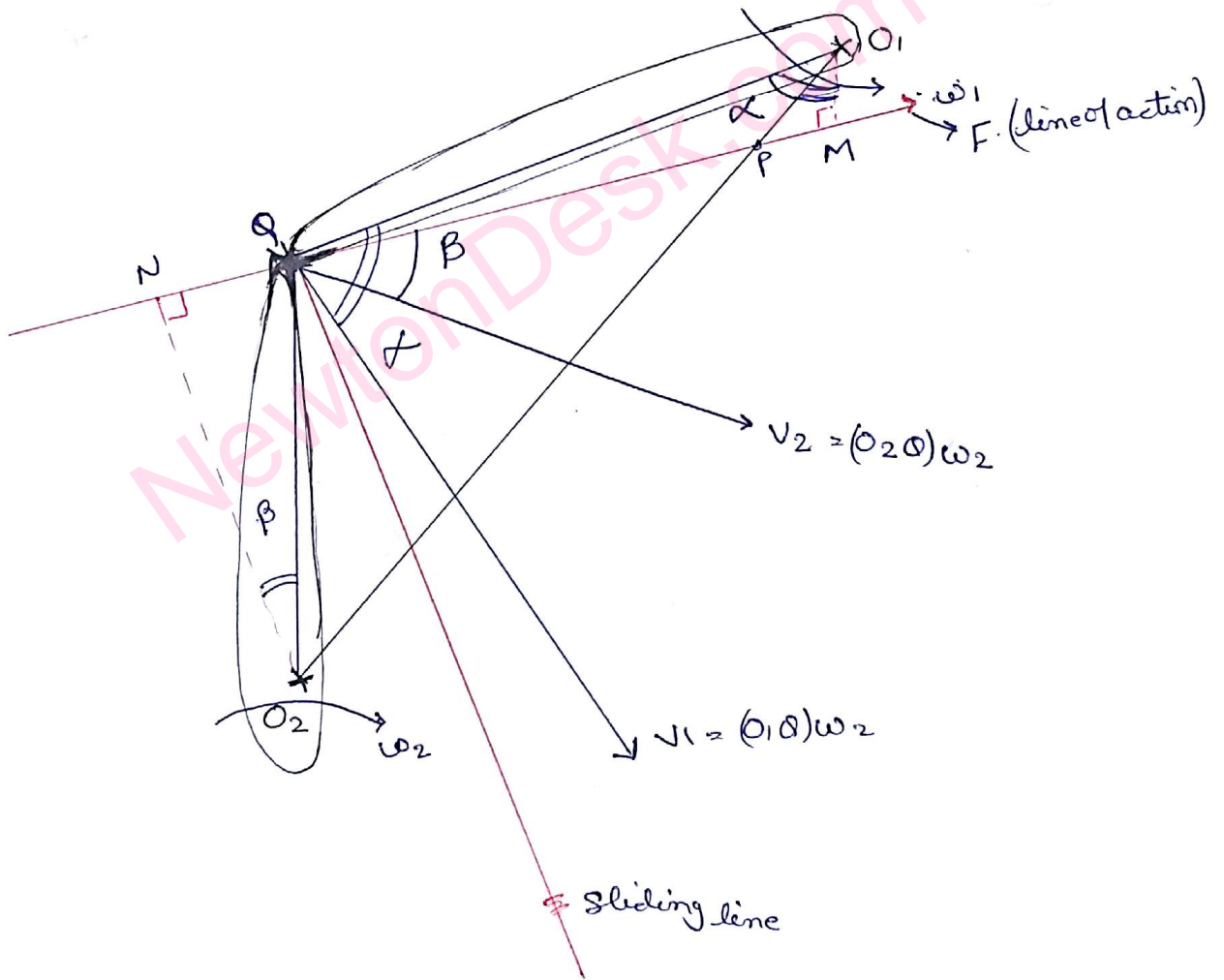
* Angle b/w line of action & Common tangent at P

$$\phi \in 20^\circ - 25^\circ$$

Most imp. Parameters



Law of Gearing :-



For Proper Contact

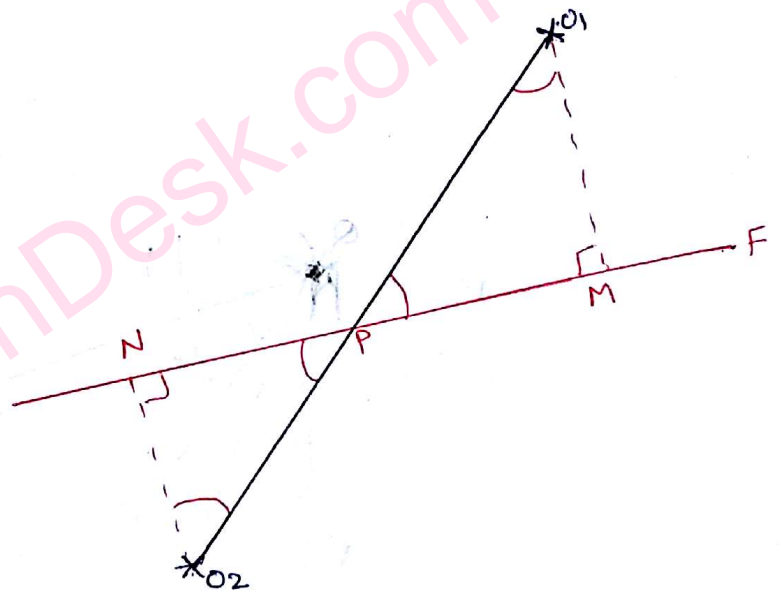
$$V_1 \cos \alpha = V_2 \cos \beta$$

$$(\cancel{O_1O}) \omega_1 \frac{O_1M}{\cancel{O_1O}} = (\cancel{O_2O}) \omega_2 \frac{O_2N}{\cancel{O_2O}}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}} \quad **$$

$$\text{In } \Delta O_1PM \sim \Delta O_2PN$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}$$



If these two Body are Gear :

$$\frac{\omega_1}{\omega_2} = \text{Constant}$$

$$\frac{O_2P}{O_1P} = \text{Constant}$$

} O_1, O_2 - Already fixed
} P - Fixed Point

P \rightarrow fixed Point

" Line of Action must Always pass through the fixed Point (Pitch Point) on the line joining the centre of rotation of gears.

For a Body to be a Gear :



Line of Action must always pass through P



Line of Action Common normal at ϕ .



"Mating Profile must be designed in such a way \otimes , such that always Law of gearing is satisfied"



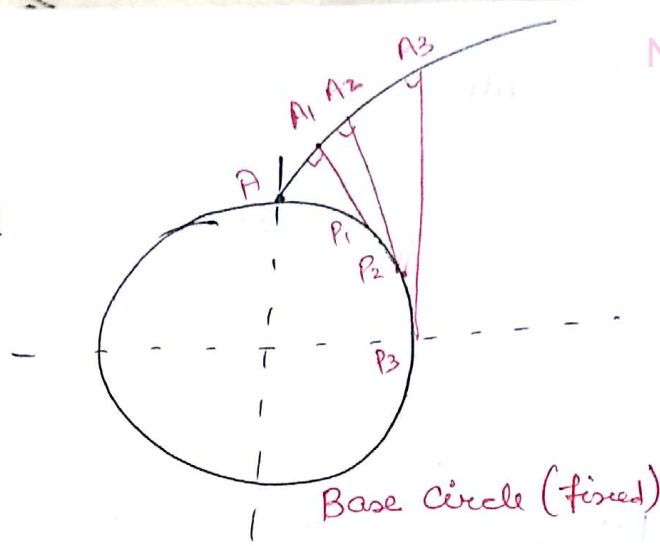
Conjugate Profiles

Involute

Cycloidal

In Reality :

$\left. \begin{matrix} AP_1 \\ P_1 P_2 \\ P_2 P_3 \\ \vdots \end{matrix} \right\} \rightarrow 0$
 Differential

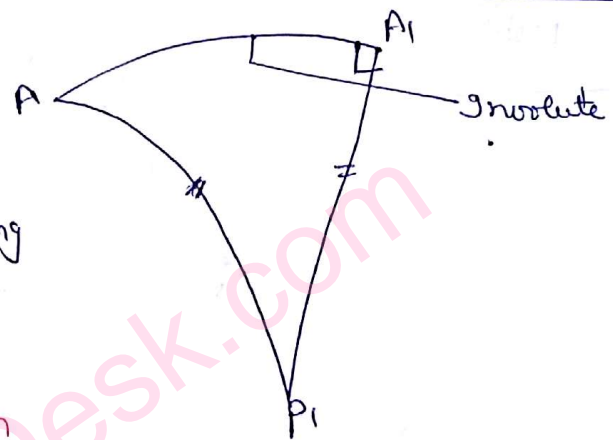


$Arc(AP_1) = P_1 A_1$

$Arc(AP_2) = P_2 A_2$

$Arc(AP_3) = P_3 A_3$

Now profile shown is
get into straight

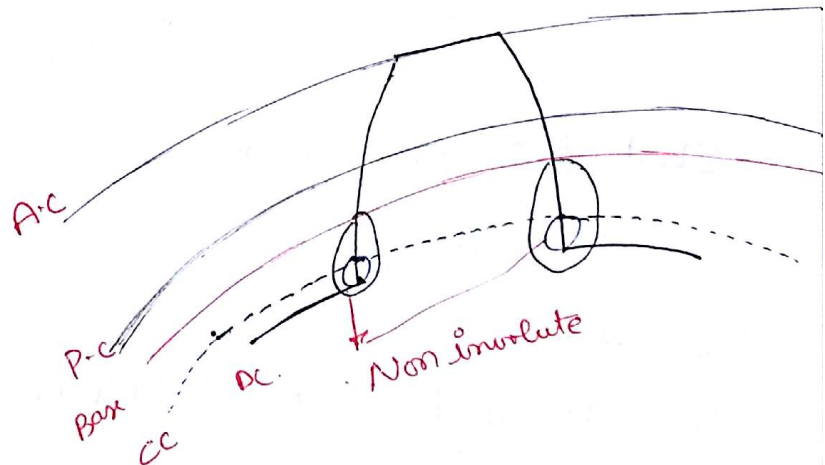


Differential Position of circle having
Centre P_1 & Radius $P_1 A_1$

"Normal drawn at any Point on
Involute curve will become tangent to its
base circle automatically."

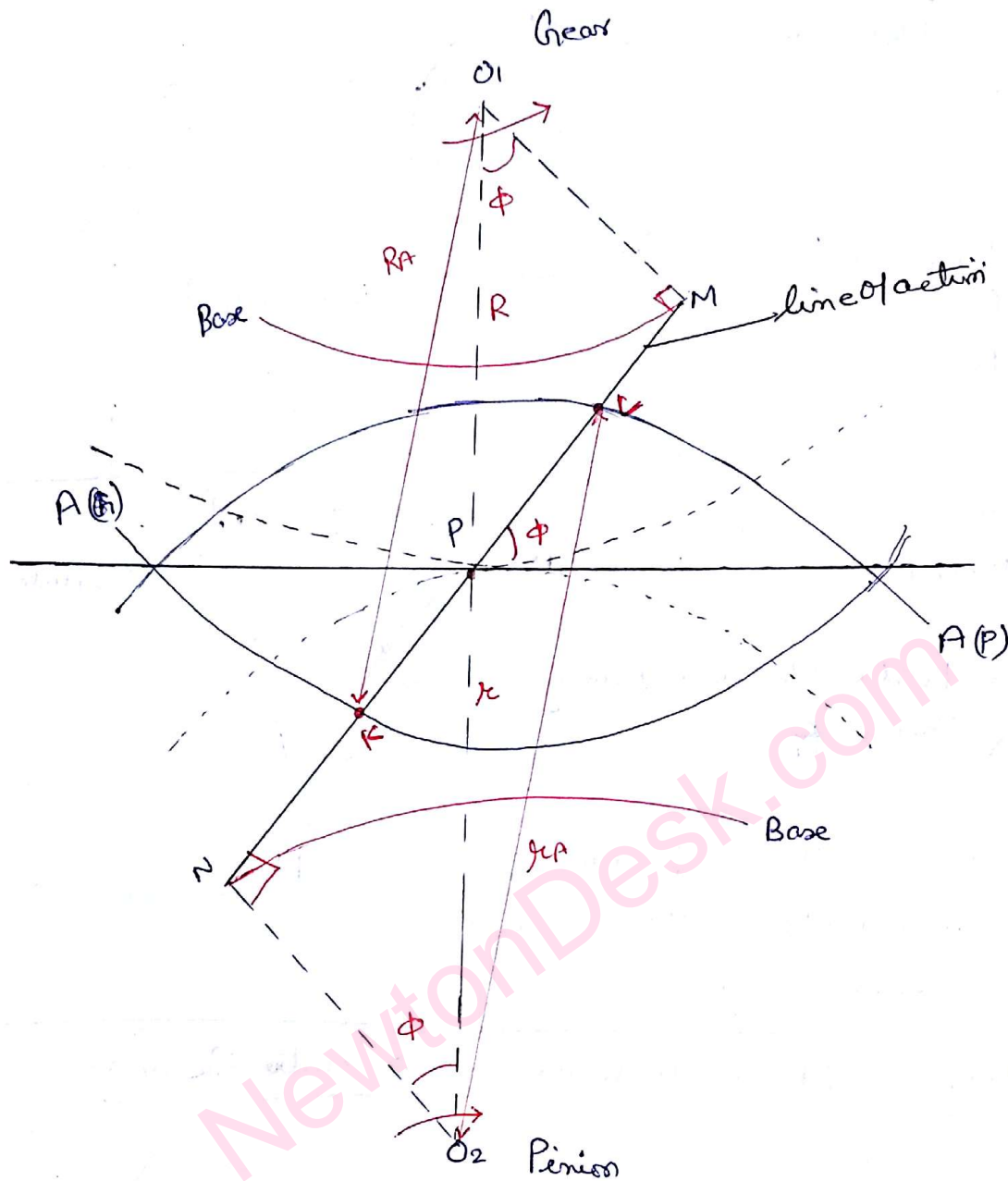
Actual Position of Base circle in an Involute Gear :-

gn. external gear
if $R_{base} \downarrow$
and Pressure angle \uparrow
(20-25°)



→ Non-Involute Position in an
external gear can never be
eliminated — (Reality)

Analysis of Involute Gears :-



Start of engagement : K

End of engagement : L

Line of Action

- 1) Pass through Line of Action or (Pitch point)
- 2) Tangent to both of Base Circle

→ Point of Contact is changing but line of action is not changing.

Hence, $\phi = \text{Constant}$

→ Point of Contact is travelling along the line of action.

Locus of Q (Point of Contact) → Straight Lines

Time interval in which Q is travelling from start to end of engagement.



One engagement Period

and the distance travelled by Q in this period.



Path of Contact

KP

Path of Approach

KL

Path of Recess.

$$O_1M = R \cos \phi$$

$$PM = R \sin \phi$$

in $\triangle O_1KM$

$$R_A^2 = R^2 \cos^2 \phi + (KP + R \sin \phi)^2$$

$$KP = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi$$

↓
Path of Approach

— (1)

Similarly

$$P_L = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi \quad \text{--- (2)}$$

↓
Path of Recess

Sum of eqⁿ (1) & (2) is Path of Contact.

Arc of Contact :

When the point of contact is travelling from the start of engagement & end of engagement, the distance ~~trav~~ travelled by pinion and gear along their pitch circles, in this period (one engagement period) is known as Arc of Contact.

$$\text{Arc of Approach} = \frac{\text{Path of Approach}}{\cos \phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of Recess}}{\cos \phi}$$

$$\text{Arc of Contact} = \frac{\text{Path of Contact}}{\cos \phi} \quad \text{**}$$

Arc of Contact \rightarrow Travel of Pinion / Gear along their Pitch Circle in one engagement period.

$$\text{Contact Ratio} = \frac{\text{Arc of Contact}}{P_c}$$

min > 1
Contact Ratio is always > 1

\downarrow
No. of Pairs engaged in one engagement period.

Contact Ratio lie b/w (1.2 - 1.8)

fore.g

Contact Ratio \neq 1.32

2nd Representation

It means, one pair is engaged in full engagement period, But in 32% time, w/ engagement period, along with this pair one more pair is engaged in one engagement period.
its average value comes out to be 1.32.

Pb. 47]

$$t = 24$$

$$T = 36$$

$$m = 8 \text{ mm}$$

$$\phi = 20^\circ$$

$$\text{Addendum (each)} = 7.5 \text{ mm}$$

$$N_p = 450 \text{ rpm}$$

$$\frac{N_G}{N_p} = \frac{t}{T} = \frac{24}{36}$$

$$N_G = 300 \text{ rpm}$$

Gear: $R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144 \text{ mm}$

$$R_A = 144 + 7.5 = 151.5 \text{ mm}$$

$$\text{Pitch} = r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96 \text{ mm}$$

$$r_A = 96 + 7.5 = 103.5 \text{ mm.}$$

i) Path of Contact:

$$KL = KP + PL$$

$$\cong \left\{ \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi \right\} + \left\{ \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi \right\}$$

$$KL = (\quad) \text{ mm} + (\quad) \text{ mm}$$

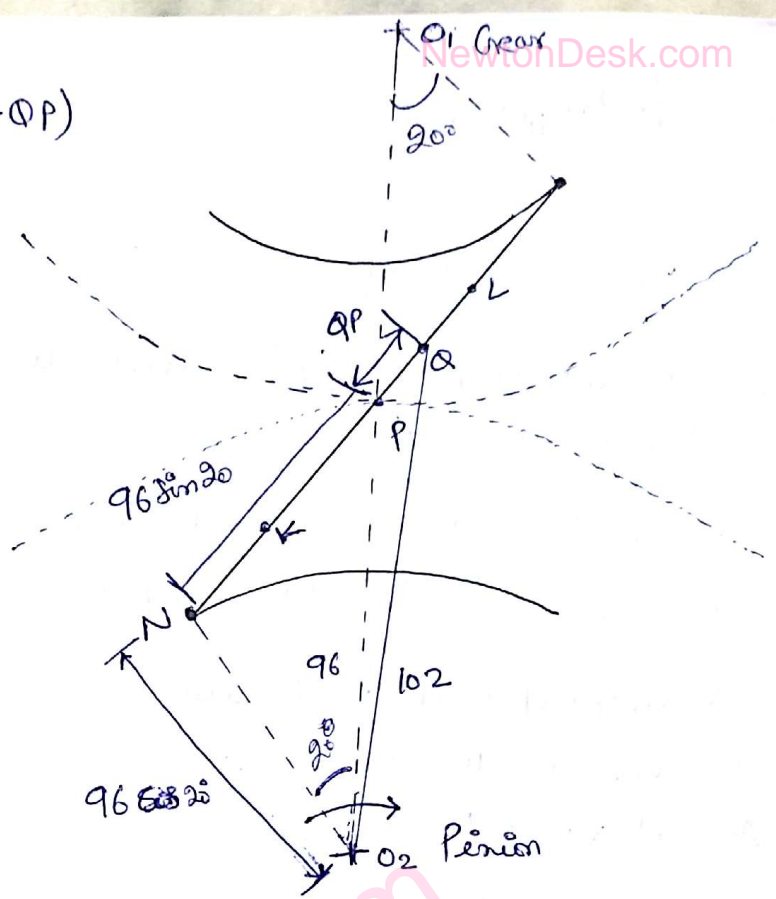
$$\text{Arc of Contact} = \frac{KL}{\cos 20^\circ} = (\quad) \text{ mm}$$

$$\text{i) (Angle) Pitch} = \frac{\text{Arc of Contact} \times \frac{180}{\pi}}{96} = (\quad)^\circ$$

$$\begin{aligned} \text{ii) } V(\text{sliding}) &= (\omega_1 + \omega_2) \cdot OP \\ &= \frac{2\pi}{60} (300 + 450) \cdot OP. \quad - \textcircled{1} \end{aligned}$$

$$(102)^2 = (96 \cos 20^\circ)^2 + (96 \sin 20^\circ + QP)^2$$

$$QP = ?$$



NewtonDesk.com

Interference :- due to $R_A \geq R_{KA} \uparrow$

If $r_A > O_2M$

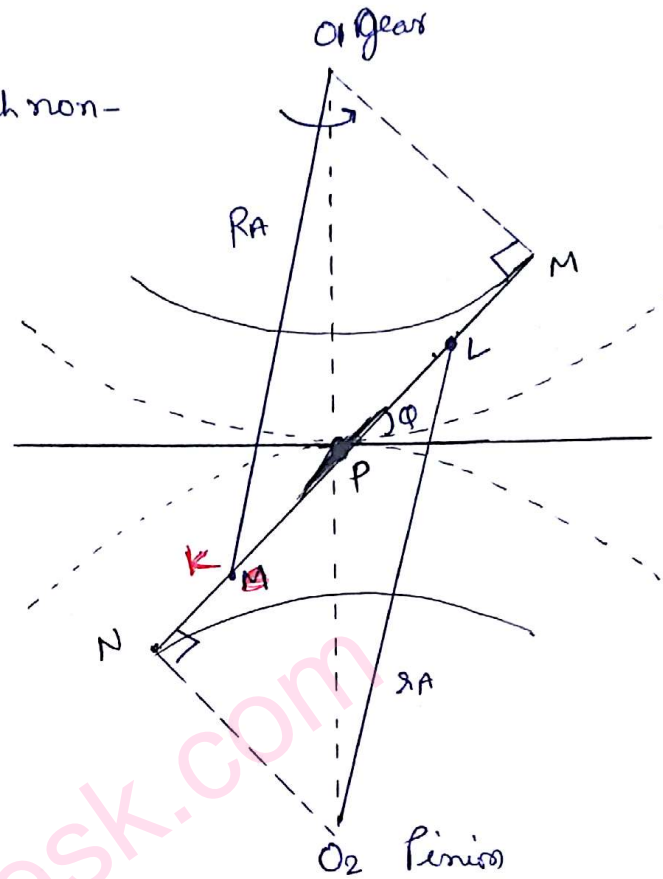
- ⇒ Involute Tip of the pinion will touch non-Involute flank Position of gear.
- ⇒ Involute → to non involute Connection
- ⇒ Law of gearing is not be satisfied.

Involute tip of the pinion will remove some material from non-Involute flank Position of Gears.

(This Removal of the material is a Process called undercutting.)

Similarly

$R_A > O_1N$



Last Safety Points of K & L are (M) and (N)

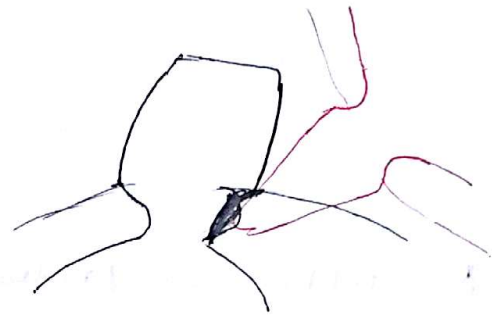
Which we can say Critical Point / Interference Point

Methods to Prevent Interference :

i) Under Cut Gears :

Under cutting is provided by the cutting tool at the time of manufacturing

Strength of tooth is less at root.



Application
~~Limitation~~ → used in low Power Transmission

ii) if Pressure Ratio (ϕ) is increased.

If ϕ is increased by decreasing r_{base}

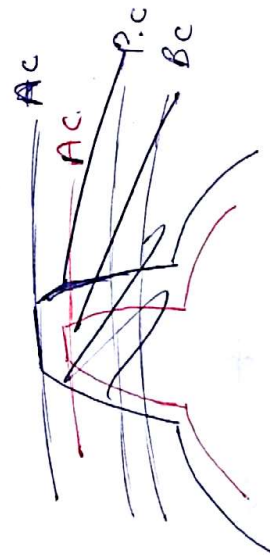
→ non involute Portion ↓

→ Interference ↓ (Main Aim)

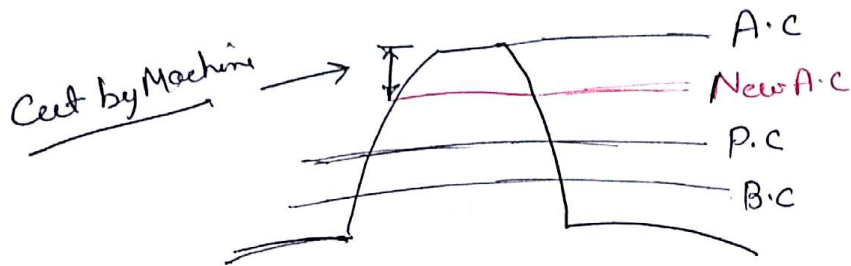
Used for
Medium Power
Transmission

Limitation of $\phi = (20^\circ - 25^\circ)$

iii) Edge Sticking at the Teeth :-



iii) By Stubbing the teeth



By Stubbing \Rightarrow Addendum \downarrow
 Addendum Circle Radius \downarrow
 Interference $\downarrow \downarrow$

By Stubbing $\rightarrow \phi \rightarrow$ Not Change

By Stubbing \rightarrow Path of Contact of \downarrow
 Arc of Contact \downarrow
 Contact Ratio \downarrow

$\min > 1$ Limitation

$$P_c = \frac{\pi D}{T}$$

\rightarrow No Change.

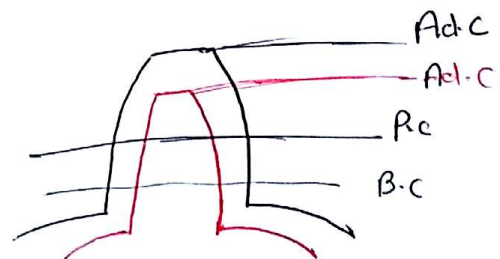
iv) Increasing number of teeth

If $T \uparrow \Rightarrow \phi \rightarrow$ No Change
 \Rightarrow Addendum \downarrow
 \Rightarrow Addendum Circle Radius \downarrow
 \Rightarrow Interference $\downarrow \downarrow$

If $T \uparrow \Rightarrow$ Arc of Contact \downarrow

$$\Rightarrow P_c = \frac{\pi D}{T} \downarrow$$

\Rightarrow Contact Ratio \uparrow



Velocity Ratio:

$$\frac{\omega_p}{\omega_s} = \frac{T}{t} > 1$$

$$\frac{\omega_s}{\omega_p} = \frac{t}{T} < 1$$

Gear Ratio (s):

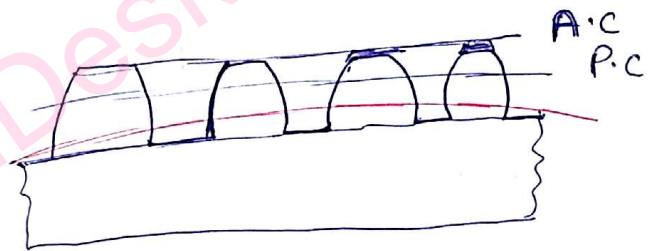
$$\boxed{G = \frac{T}{t}}^{**} \text{ Always } \geq 1$$

Rack:

Gear of infinite Pitch circle Diameter (P_c)

→ Biggest Gear

Base circle seems to be straight but it is not straight. it is ~~straight~~ circle radius of infinite Dia



Note

A → Fractional Addendum for 1 mm module in order to avoid interference.

$\left. \begin{matrix} A_p \\ A_G \\ A_R \end{matrix} \right\}$

Therefore, Addendum Required in order to avoid interference.

$$\begin{aligned} &= m A \quad (\text{where } m \text{ is module}) \\ &= m A_p \\ &= m A_G \\ &= m A_R \end{aligned}$$

for examples:

$$\left. \begin{aligned} \text{Addendum} &= 7.5 \text{ mm} \\ \text{module (m)} &= 8 \text{ mm} \end{aligned} \right\}$$

$$mA = 7.5$$

$$A = \frac{7.5}{8} < 1 \text{ So it is Stub}$$

Involute Gear System:

Full depth Involute: $(14\frac{1}{2}, 20^\circ)$

Addendum = Standard Addendum
= one module value

$$mA = 1$$

$$\boxed{A = 1}$$

$$A_P = 1$$

$$A_G = 1$$

$$A_R = 1$$

Stub Involute $(20^\circ, 25^\circ)$

Addendum $<$ Standard Add.

$$mA < 1$$

$$\boxed{A < 1}$$

$$\boxed{\begin{array}{l} A_P \\ A_R \\ A_G \end{array} < 1}$$

→ 20° Involute Stub

Best Choice

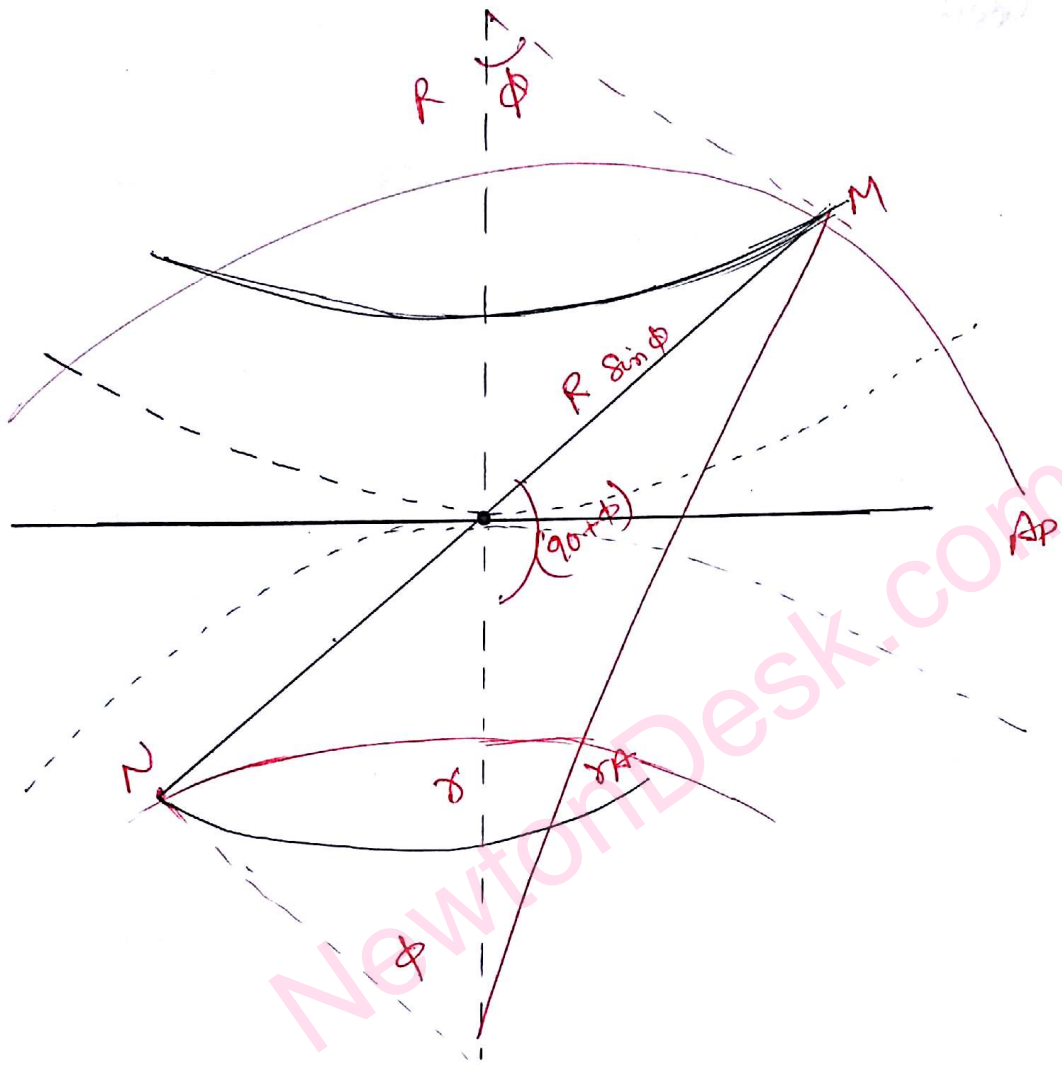
- 1) Lesser Interference
- 2) Min no. of teeth, requirement is less.
- 3) Cost is less
- 4) Stronger tooth

NewtonDesk.com

Minimum Number of teeth requirement on

Pinion or Gear, in order to avoid Interference

$$(K-L) + \text{Safety limit}$$



Cos Rule $\rightarrow R$

$$r_A^2 = r^2 + R^2 \sin^2 \phi - 2(r)(R \sin \phi) \cdot \cos(90 + \phi)$$

$$= r^2 \left(1 + \left(\frac{R}{r}\right)^2 + 2 \left(\frac{R}{r}\right) \sin^2 \phi \right)$$

$$r_A = r \sqrt{1 + G(G+2) \sin^2 \phi}$$

$\rightarrow r_A - r = \text{Add. Penion}$

$$= r \left[\sqrt{1 + G(G+2) \sin^2 \phi} - 1 \right]$$

$$= \frac{r \sin \phi}{2} \left(\sqrt{1 + G(G+2) \sin^2 \phi} - 1 \right) = r \sin \phi AP$$

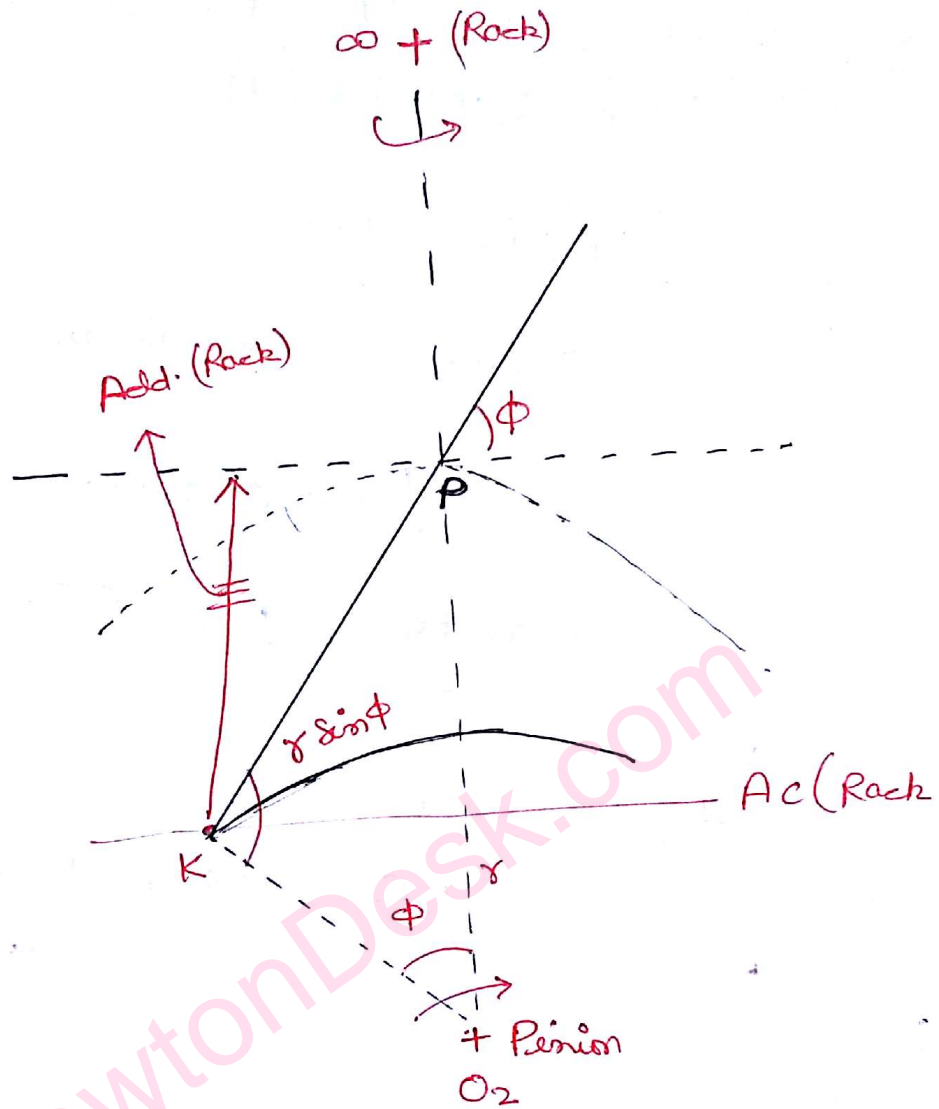
$$t_{\min} = \frac{2AP}{\sqrt{1 + G(G+2) \sin^2 \phi} - 1}$$

Similarly

$$T_{\min} = \frac{2AG}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1} *$$

Minimum number of teeth requirement on the Pinion in order to avoid interference in involute Rack & Pinion arrangement :

Arrangement :



$$\sin \phi = \frac{\text{Add. (Rack)}}{r \sin \phi}$$

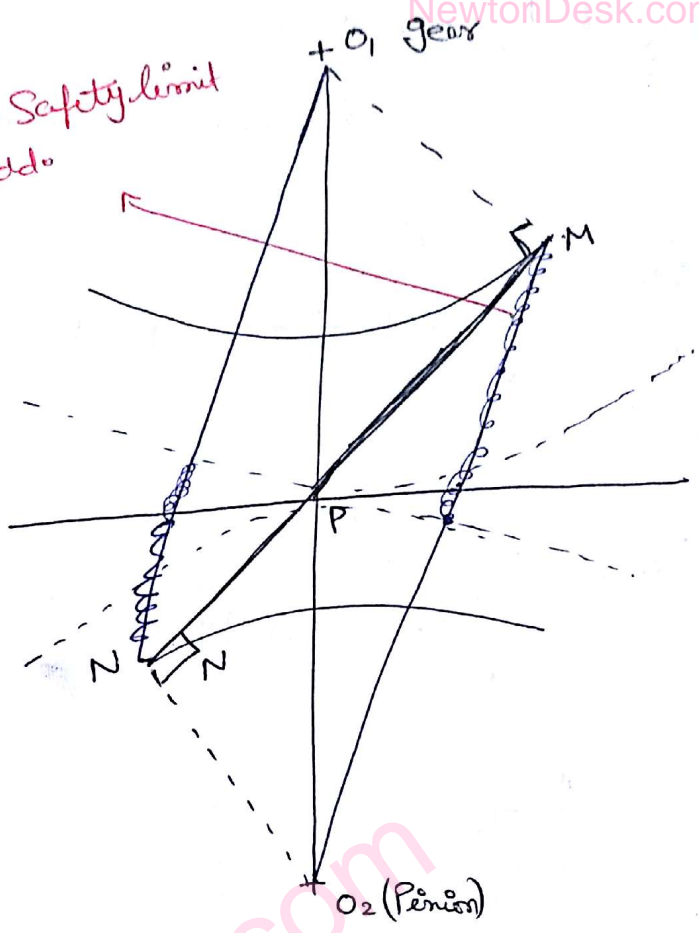
$$\text{Add Rack} = r \sin^2 \phi$$

$$\cancel{r \sin} = \frac{m t_{\min} \sin^2 \phi}{2} = \cancel{r} m A R$$

$$t_{\min} = \frac{2 A R}{\sin^2 \phi} \times \times$$



Maximum Safety limit
for Pinion Addo
to Protect
Interference



26 Gear / Pinion

↳ Addendum same.

1) first Gear must be safe

$$T_{min} = \frac{2A_a}{\sqrt{1 + \frac{1}{a} \left(\frac{1}{a} + 2\right) \sin^2 \phi} - 1}$$

2) Pinion will automatically safe.

$$t_{min} = \frac{T_{min}}{a}$$

If Examiner ask to Wah abt.
 $\sigma_r = 3$

28 → 30
toget
③

$\frac{28}{3} = 9.9$
⑩

ii) 26 Pinion/Gear

→ Gear Adds - Different

$$G = 3$$

$$T_{\min} = \frac{2AG}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1} \rightarrow (40) - 42$$

$$t_{\min} = \frac{2Ap}{\sqrt{1 + G(G+2) \sin^2 \phi} - 1} \rightarrow (13) - (14)$$

$$\frac{T_{\min}}{t_{\min}} = G$$

NewtonDesk.com

Qn) 48

$$G = 3$$

$$A_p = F A = 1 \quad \phi = 20^\circ \quad T_{\min} = ?$$

$$T_{\min} = \frac{2 A G}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1} = 45 \times$$

$$T_{\min} = \frac{45}{3} = 15 \times$$

$$G = 3 \rightarrow T_{\min} = 12 \times 36 = 36 \times$$

→ Yes interference occurs

$$T_{\min} = 36 = \frac{2 A G}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1} =$$

$$A G = 0.8$$

20% Stabbing Ad

Qn



$$T_{\min} = 36 = \frac{2 A G}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$$

$$\phi = ?$$

Pb Q6

$$\phi = 20^\circ$$

$$m = 10 \text{ mm}$$

$$A_p = A_g = 1$$

$$T = 50 \quad] \quad G = \frac{50}{13}$$

$$t = 13$$

$$i) \quad T_{min} = \frac{2A_g}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi - 1}} = 60$$

(yes)

$$ii) \quad T_{min} = 50 = \frac{2A_g}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi - 1}}$$

$$\phi = ?$$

Qm

$$G = 4$$

$$A_p = A_g = 1$$

$$\phi = 20^\circ$$

find $T_{min} = ?$

$$T_{min} = \frac{2A_g}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi - 1}}$$

$$= \frac{64}{2} \quad (64)$$

$$t_{min} = \frac{6g}{4} = 15.5 \quad (16)$$

$$G = \frac{64}{15} \quad (4)$$

~~T~~ T_{min} for pinion use कराना जरूरी है .

$G =$ Gear Ratio के लिये ।

As Gear safe pinion safe

check pinion also

Effect of the Centre Distance Variation Due to Vibrations

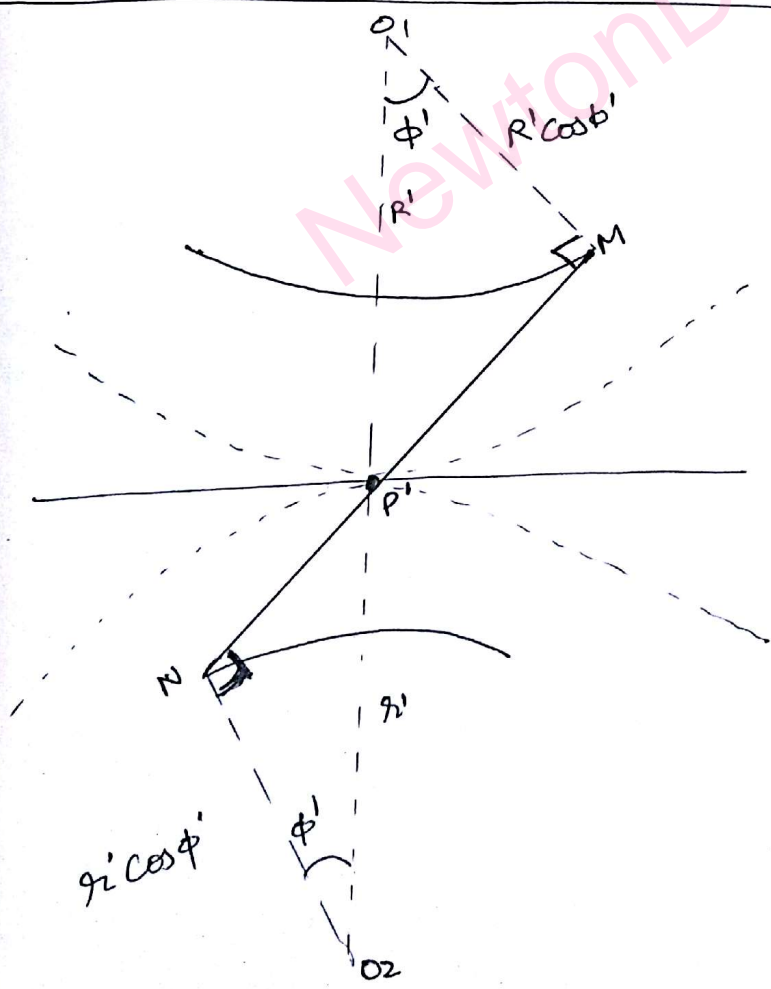
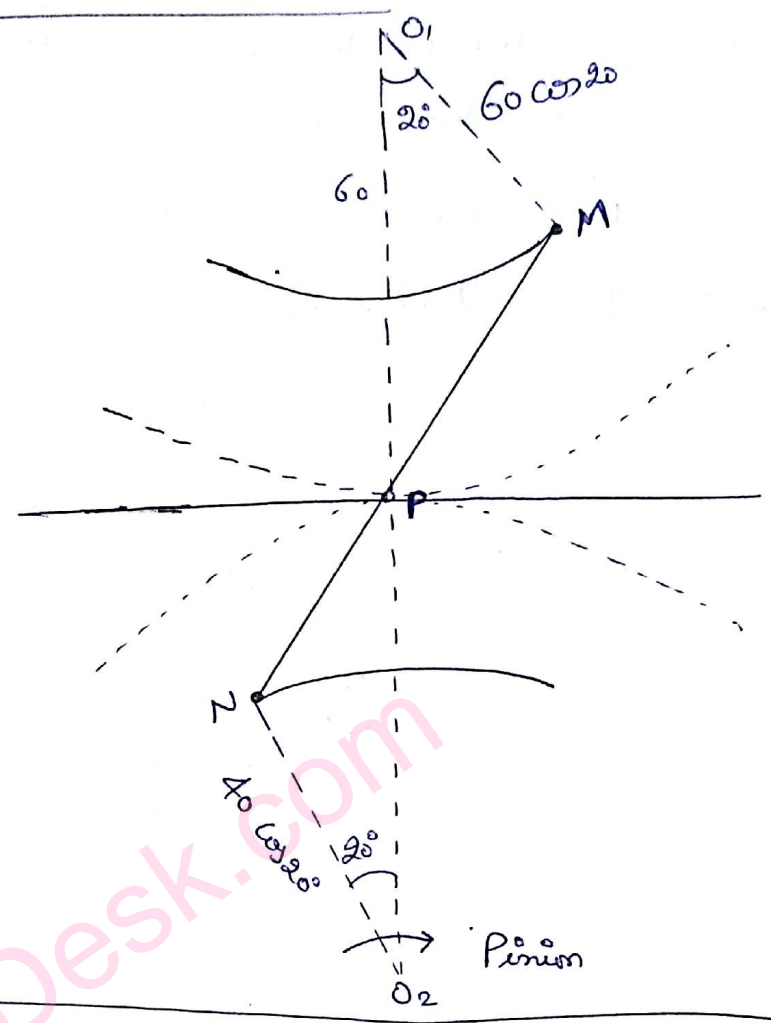
On the performance of involute gears :-

Centre distance = 100 mm
 $R = 60$ mm
 $r = 40$ mm
 $\phi = 20^\circ$

At a moment

Centre distance is ϕ by 2%

100 mm
 \downarrow
 102 mm



$$R' \cos \phi' = 60 \cos 20$$

$$r' \cos \phi' = 40 \cos 20$$

$$\frac{(R' + r') \cos \phi' = 100 \cos 20}{\downarrow 102}$$

$$\cos \phi' = \frac{100 \cos 20}{102}$$

$\phi = ?$

Bcz of Vibration :

- Center distance is Changing
- Pc Changing
- P Changing
- ϕ Changing

$k_{Base} \rightarrow$ No Change

$$\frac{\omega_1}{\omega_2} = \frac{C_2 N}{C_1 M}$$

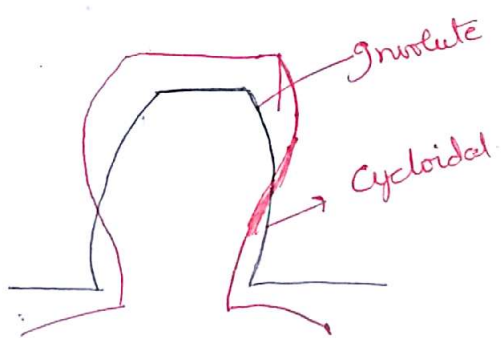
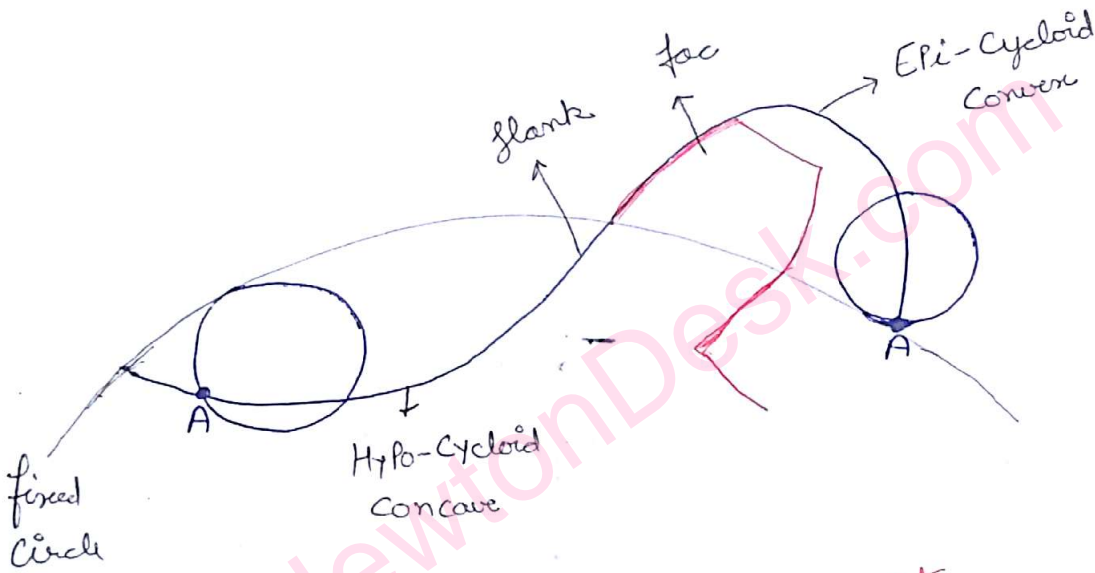
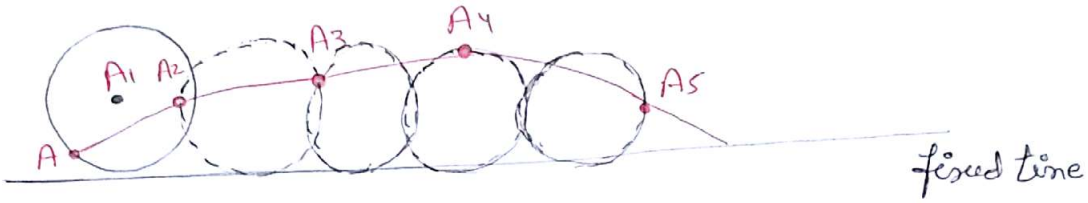
↓
Constant

After Vibration
Constant Velocity
Ratio obtained

NewtonDesk.com

Cycloidal Profile (By Natural Conjugate) : NewtonDesk.com

" It is defined as locus of the point on the circumference of the circle, which rolls without slipping, on the fixed straight line.



1) Per tooth Cost is more.

Overall Cost of Gear-lens (No machining or extra removal of tooth bottom surface)
(Almost equal)

Interference is absent

2) Flanks wide \rightarrow Stronger teeth

3) Convex-Concave Connection
 \rightarrow Less wear & tear

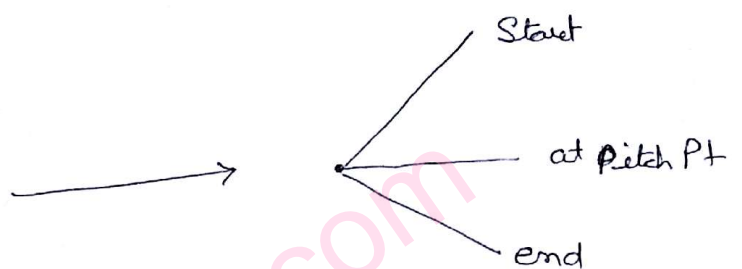
Life is 5 times more than involute (1000 years)

ϕ Changing

$\phi_{max} \rightarrow$ At start of engagement

0 \rightarrow At Pitch Point

$\phi_{max} \rightarrow$ at end of engagement
(in Reverse dirⁿ)



Note:

Power Component of force: $F \cos \phi$

For example

The Power Component of force is 0.94 times the Normal Thrust
 \downarrow
Force

It means

$$F \cos \phi = 0.94 F$$

$$\cos \phi = 0.94$$

So we can calculate value of ϕ

i.e. Pressure angle

Torque

$$T_{\text{Pinion}} = F \cos \phi \times r$$

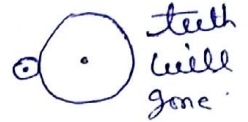
$$T_{\text{Gear}} = F \cos \phi \times R$$

$$\text{Power} = T \times \omega$$

Gear Trains

Gear Train is Combination of Gears.

→ Why it is Required?
↓



- i) Large Centre distance
- ii) Very high / very Low Requirement of velocity Ratio $\rightarrow \frac{\omega_1}{\omega_2} = \frac{10}{1} = \frac{R_2}{R_1}$
- iii) Multiple velocity Ratios are required

Any gear train is Combination of following
↓

- i) Main Driver
 - ii) Main Driven
 - iii) Intermediate Gear
 - iv) Arm. (for epi-cyclic Gear Train) used in Differential Box.
- Max. min

$$\frac{\omega_{\text{main Driver}}}{\omega_{\text{main Driven}}} = \text{Speed Ratio (SR) of Gear Train} \quad **$$

$$\frac{\omega_{\text{main DVN}}}{\omega_{\text{main DVR}}} = \frac{1}{S.R} = \text{Train Value} \quad **$$

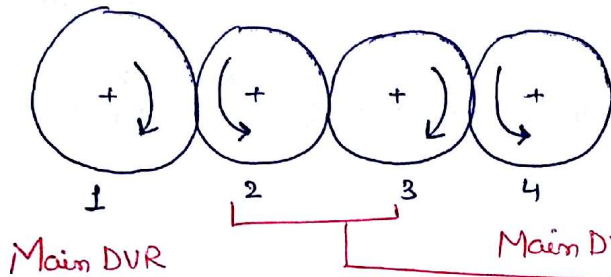
Simple Gear Train :



Every shaft is having only one gear in else.

$$m_1 = m_2 = m_3 = m_4$$

$m_{all} \rightarrow$ Same



No Contribution in Speed Ratio

Intermediate Gear or Idlers

$$(1,2) \quad \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad \text{--- ①}$$

$$(2,3) \quad \frac{\omega_2}{\omega_3} = \frac{T_3}{T_2} \quad \text{--- ②}$$

$$(3,4) \quad \frac{\omega_3}{\omega_4} = \frac{T_4}{T_3} \quad \text{--- ③}$$

gf NO. of Idlers
 → even → Dirⁿ opposite
 → odd → dirⁿ same

Product of ①, ②, ③

$$\boxed{\frac{\omega_1}{\omega_4} = \frac{T_4}{T_1}} \quad \text{--- ** Speed Ratio}$$

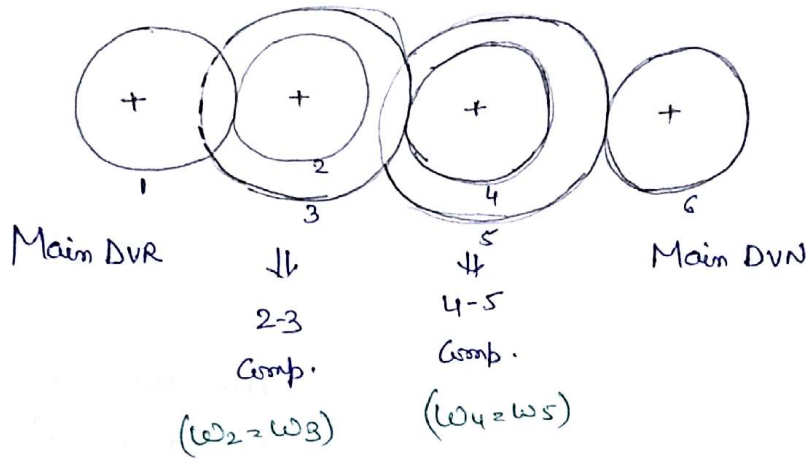
Gear telhich mating having same module always

Pinion (small one) is taken as Driver.

Compound Gear Train :

At least one of the intermediate shaft must have at least one gear in use.

$$\left. \begin{aligned} m_1 &= m_2 \\ m_3 &= m_4 \\ m_5 &= m_6 \end{aligned} \right\}$$



$$\begin{aligned} \text{DVR} &= (1, 3, 5) \\ \text{DVN} &= (2, 4, 6) \end{aligned}$$

$$(1-2) \quad \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

$$(3-4) \quad \frac{\omega_3}{\omega_4} = \frac{T_4}{T_3} \quad \text{--- (2)}$$

$$(5-6) \quad \frac{\omega_5}{\omega_6} = \frac{T_6}{T_5} \quad \text{--- (3)}$$

Product of (1) (2) (3)

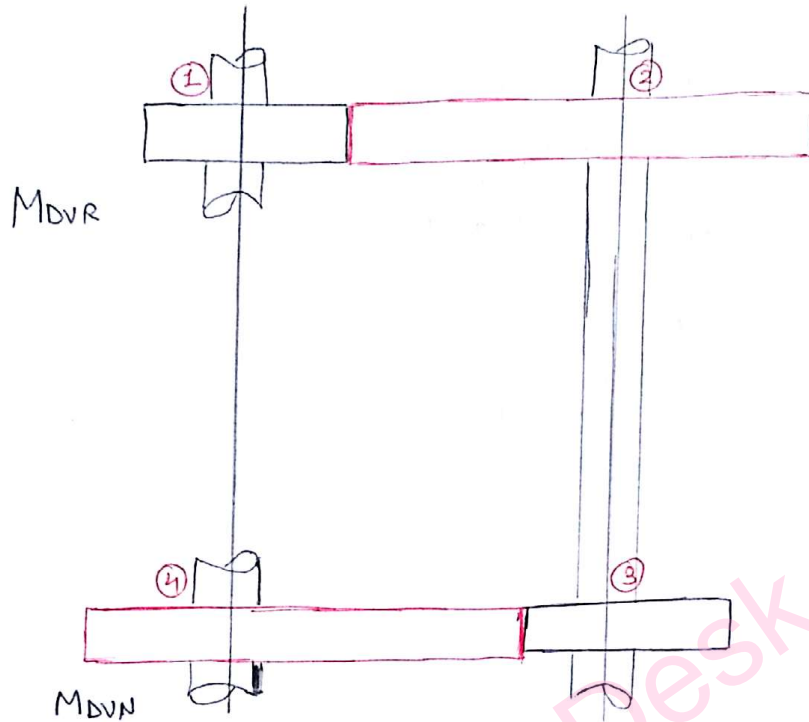
$$\frac{\omega_1}{\omega_6} = \text{S.R} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5} \quad \begin{matrix} *** \\ ** \\ * \end{matrix}$$

$$\text{Speed Ratio} = \frac{\text{Product of the no. of Teeth on DVN}}{\text{Product of the no. of teeth on DVR}}$$

Reverted Gear Trains :-



→ That Compound Gear Train which is used to connect co-axial shaft.



$$m_1 = m_2 = m$$
$$m_3 = m_4 = m'$$

$$DVR = (1, 3)$$
$$DVN = (2, 4)$$

$$S.R = \frac{\omega_1}{\omega_4} = \frac{T_2 T_4}{T_1 T_3} \quad **$$

NOTE :-

If in the Problem of Reverted gear train, Speed reduction is given same

Then Take

$$\frac{T_2}{T_1} = \frac{T_4}{T_3}$$

A General Concept :-

$$r_1 + r_2 = r_3 + r_4$$

$$r = \frac{mT}{2}$$

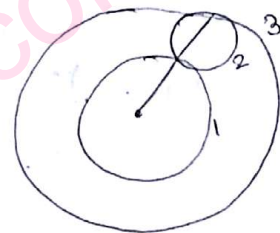
$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$m(T_1 + T_2) = m'(T_3 + T_4)$$

if all gears have same Modules; (All m same)

$$T_1 + T_2 = T_3 + T_4$$

For example



$$r_1 + 2r_2 = r_3$$

Mod same

$$T_1 + 2T_2 = T_3$$

Q. 44)

Given: $m = 2 \text{ mm}$
 $m' = 3 \text{ mm}$
 $\omega_4 < \frac{\omega_1}{12}$

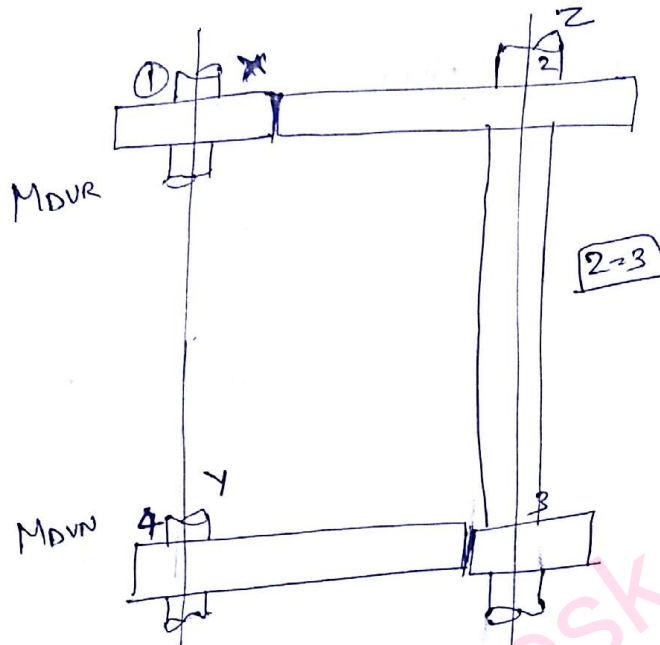
$T_2 = ?$

$T_4 = ?$

$T_1 = ?$

$T_3 = ?$

$$T_1 = T_3 = 24 \text{ g\u00fcnsen}$$



$$\frac{\omega_1}{\omega_2} > 12$$

$$\frac{T_2 T_4}{24 \times 24} > 12$$

$$T_2 T_4 > 6912 \text{ --- (1)}$$

$$m(T_1 + T_2) = m'(T_3 + T_4)$$

$$2(24 + T_2) = 3(24 + T_4)$$

\downarrow \downarrow
 T_1 T_3

$$48 + 2T_2 = 72 + 3T_4$$

$$3T_4 = 2T_2 - 24$$

$$T_4 = \frac{2}{3}(T_2 - 12) \text{ --- (2)}$$

$$\frac{2}{3}(T_2 - 12)T_2 > 6912$$

$$T_2^2 - 12T_2 - 10638 \Rightarrow 0$$

$$T_2 > 108$$

Assume

$$T_2 = 109 \Rightarrow T_4 = \frac{2}{3}(109 - 12)$$

$$T_4 = 64.66$$

Hit and trial Method;

Assume $T_4 = 65$

$$T_3 = 24 - 1 = 23$$

$$m(T_1 + T_2) = m'(T_3 + T_4)$$

$$2(24 + T_2) = 3(23 + 65)$$

$$T_2 = 108$$

$$\left. \begin{array}{l} T_1 = 24 \\ T_2 = 108 \\ T_3 = 23 \\ T_4 = 65 \end{array} \right\}$$

$$\frac{w_1}{w_4} \cdot \frac{T_2 T_4}{T_1 T_3} = \frac{108 \times 65}{24 \times 23} = 12.718$$

Central distance

$$= (r_1 + r_2)$$

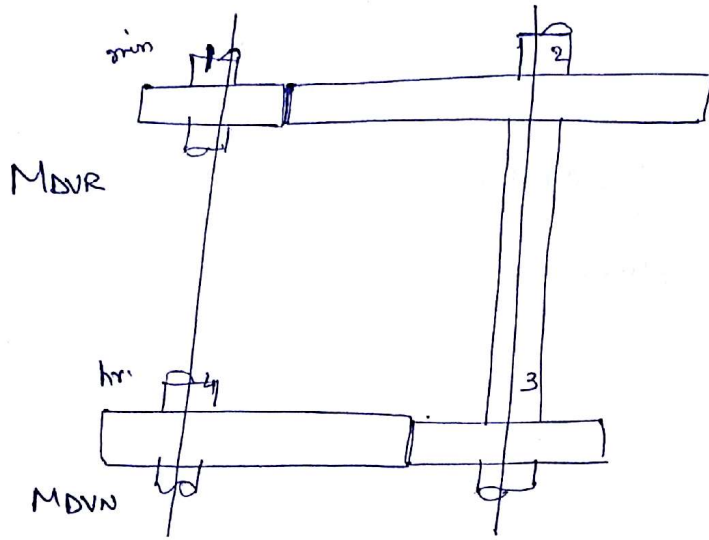
$$= \frac{mT_1}{2} + \frac{mT_2}{2}$$

$$= \frac{2}{2}(24 + 108)$$

$$= 132 \text{ mm}$$

Centre
Distance

Qn) A reverted gear train is used in a clock to drive the hour hand with help of minute hand. Find the suitable NO of the teeth for the gear. any gear should not have less than 11 teeth.



$$T_1, T_2, T_3, T_4 < 11$$

$$m_1 = m_2 = m_3 = m_4$$

Assume all module same

$$\omega_1 (T_1 + T_2) = \omega_4 (T_3 + T_4)$$

$$T_1 + T_2 = T_3 + T_4$$

$$\frac{\omega_1}{\omega_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

$$\omega_1 = \frac{2\pi}{60 \times 60} = \text{Rad/sec}$$

$$\omega_4 = \frac{2\pi}{12 \times 60 \times 60} = \text{rad/sec}$$

$$\frac{\omega_1}{\omega_4} = 12 = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$\frac{\omega_1}{\omega_4} = 12$$

all module same

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Assume } T_1 = 12$$

$$\text{Assume, } \frac{T_2}{T_1} = 4 \quad \frac{T_4}{T_3} = 3$$

$$T_2 = 48$$

$$(12 + 48) = T_3 + T_4 = 3T_3 + T_3 = 4T_3$$

$$T_3 = \frac{60}{4} = 15, \quad T_4 = 3T_3 = 45$$

$$\left. \begin{array}{l} T_1 = 12 \\ T_2 = 48 \\ T_3 = 15 \\ T_4 = 45 \end{array} \right\} \text{An } (51)$$

(11)

Answer

$$\frac{T_2}{T_1} = 6 \quad T_2 = 6T_1 = 72$$

$$12 + 72 = T_3 + T_4$$

$$84 = 3T_3$$

$$T_3 = 28$$

$$T_4 = 56$$

$$\left. \begin{array}{l} T_1 = 12 \\ T_2 = 72 \\ T_3 = 28 \\ T_4 = 56 \end{array} \right\}$$

$$\frac{T_2}{T_1} = 2$$

$$\frac{T_4}{T_3} = 6$$

$$T_2 = 2T_1$$

$$T_4 = 3T_3$$

$$~~12 + 24~~$$

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Epi-Cyclic Gear Cycle :-

DOF = 2

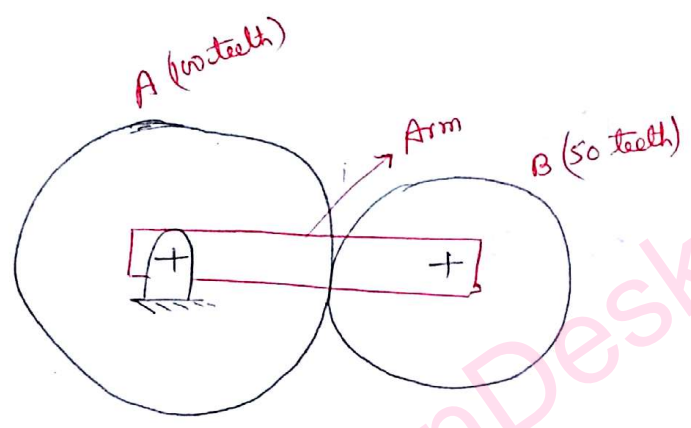
Arm speed is same for all gears

Axis Rotation

A Part from the rotation of the gears, if any gear axis is also rotating w.r. to some other axis, the train is known as Epi-Cyclic gear Train.

It may be Simple epicyclic, Compound epicyclic, Reverted epicyclic, Bevel epicyclic & so on.

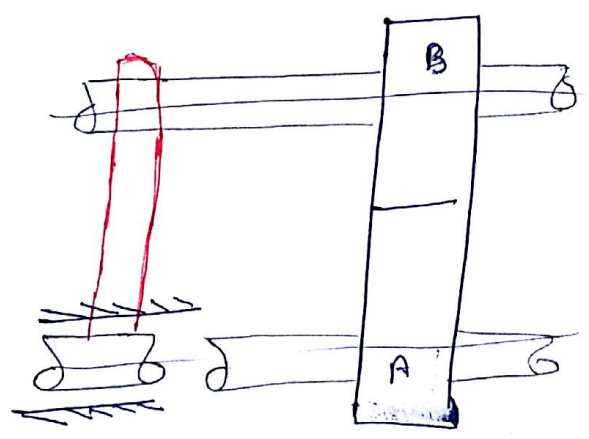
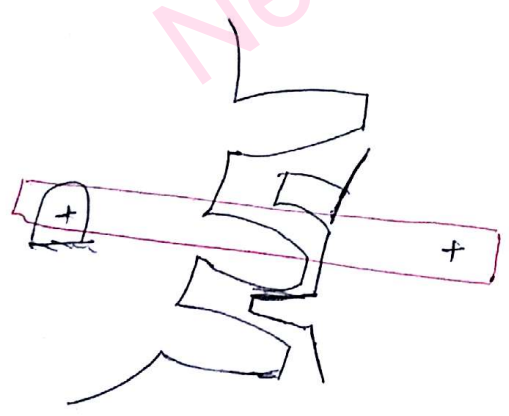
To rotate the Axis of the gear, a link is used which is known as Arm or Carrier. Arm is not a gear. it is simply a link & most imp part of epi-cyclic gear Train.



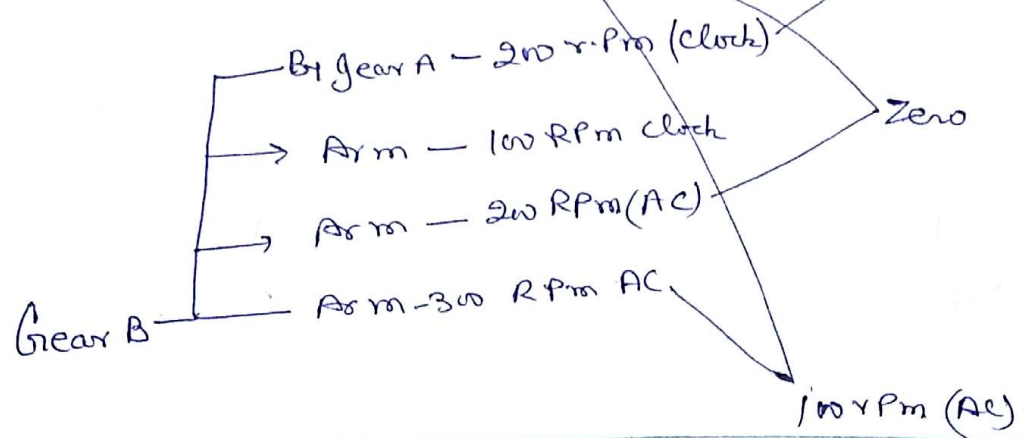
Law of gearing

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}$$

$$\omega_2 = \frac{T_1 \omega_1}{T_2}$$



Let us take Gear A \rightarrow 100 RPM (AC) NewtonDesk.com



Pb) $T_A = 20$
 $T_B = 30$
 $T_E = T_F = 10$

And All gear have same Module.

$$T_A + 2T_E = T_C$$

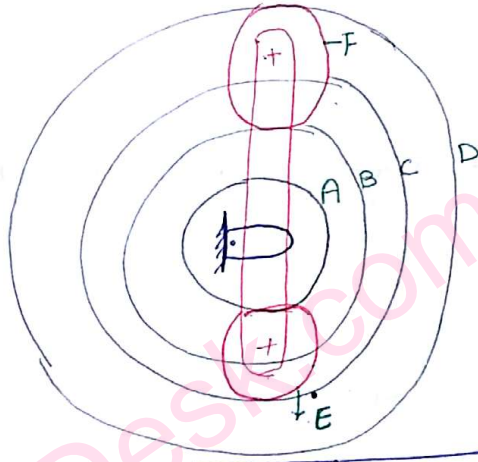
$$\downarrow$$

$$40x$$

$$T_B + 2T_F = T_D =$$

$$\downarrow$$

$$50$$



Motions	Arm	A/B <small>20/30</small>	E <small>10</small>	C <small>40</small>	F <small>10</small>	D <small>50</small>
1) Arm fixed Let gear A rotates by $+x$ rpm (clock)	0	$+x$	$-x \frac{20}{10}$	$-x \frac{20 \cdot 10}{40}$	$-x \frac{30}{10}$	$-x \frac{30}{10} \frac{10}{50}$
2) Arm free	y	$(y+x)$	$y-2x$	$y-\frac{x}{2}$	$y-3x$	$y-\frac{3x}{5}$

\nearrow Solⁿ ②

$$y - \frac{3x}{5} = 0 \quad \text{--- ①}$$

$$y = -100 \quad \text{--- ②}$$

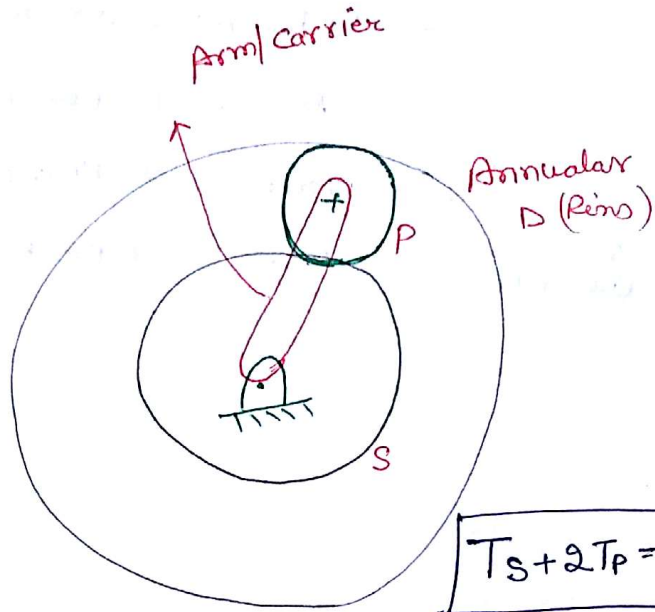
$$x = ?$$

$$y = ?$$

Planetary Gear Trains:

I Input

Sum	Ring
fixed	Input
Input	fixed



II Input

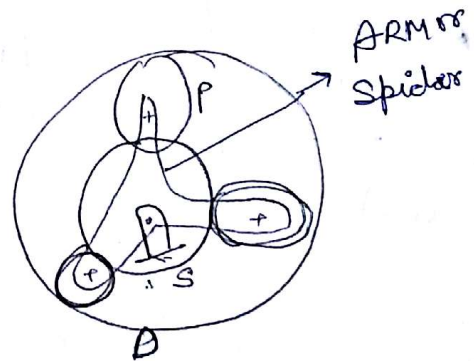
ARM

** Generally in Planetary gear train, no. of Planet gear are more than one
 ↓
 why
 ↓

- i) For the Balancing of System
- ii) For the load distribution among the Planets in high Power Transmission

Qn) 45 Pg 24

Arm (A)	S (T _S)	P (T _P)	D (T _D)
0	+x	$-\frac{x T_S}{T_P}$	$-\frac{x T_S}{T_P} \cdot \frac{T_P}{T_D}$
y	(y+x)	$y - \frac{x T_S}{T_P}$	$y - \frac{x T_S}{T_D}$

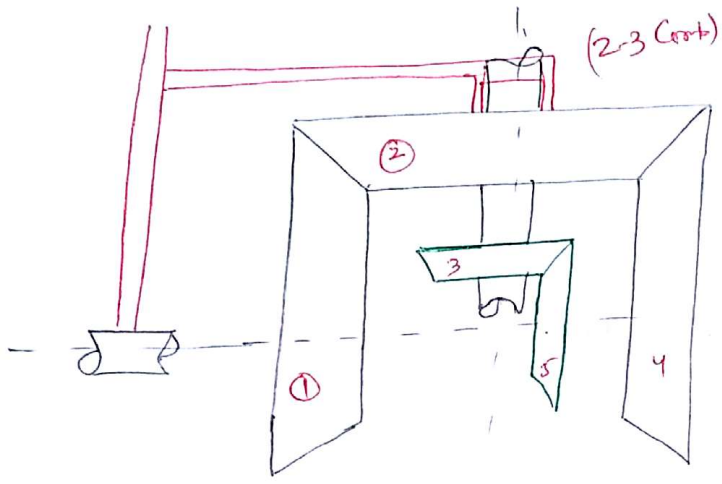


$\frac{252}{3.5} = T_D$, $T_D = 72$, $T_S + 2T_P = 72$ $N_D = 20$ $N_S = 5 N_A$

$y + x = 5y$
 $x = 4y$
 $y - \frac{x T_S}{72} = 0$

$y \left(1 - \frac{T_S}{18}\right) = 0$

$T_S = 18$



Arm	1	2/3	4	5
ω	$+x$	$\pm x \frac{T_1}{T_2}$	$\ominus x \frac{T_1 \cdot T_2}{T_2 \cdot T_4}$	$\ominus x \frac{T_1 \cdot T_3}{T_2 \cdot T_5}$
ω	$y+x$	$(y \pm x \frac{T_1}{T_2})$	$(y - x \frac{T_1}{T_4})$	$(y - x \frac{T_1 \cdot T_3}{T_2 \cdot T_5})$

Fixing/Holding Torque in an epi-cyclic Gear Train:-
Total Torque in an epicyclic gear Train;

$$\sum T = T_{input} + T_{output} + T_{fixing} = 0 \quad \text{--- (1)}$$

Power Conservation;

$$T_{input} \cdot \omega_{input} + T_{output} \cdot \omega_{output} = 0 \quad \text{--- (2)}$$

$\eta_{gear\ Train} = 1$

$$N_{in\text{put}} = +100$$

$$N_{out\text{put}} = +250$$

$$T_{in\text{put}} = +50$$

$$T_{fixing} = ?$$

$$(50) + (-20) + T_{in\text{put}} = 0$$

$$T_{out\text{put}} = -20$$

$$(50) + (-20) + T_{fixing} = 0$$

$$T_{fixing} = -30$$

$$= 30 \text{ KN-m AC}$$

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Balancing



Vibrations Causing Unbalanced

Rotating unbalance

(By rotating mass)

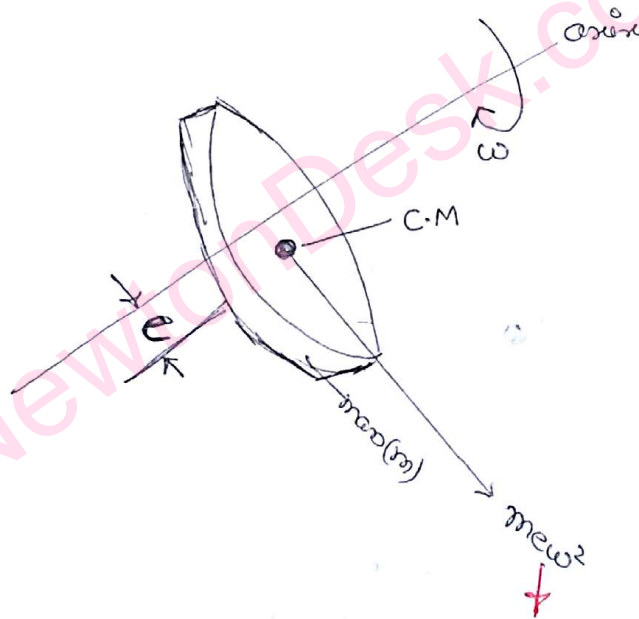
Resiprocating unbalance

(By Resiprocating mass)

Balancing

$$\sum \vec{F} = 0$$

$$\sum \vec{M} = 0$$



Always Constant in Magnitude but changing in dirⁿ.

Rotating unbalance

Static Balancing



Here we balance only forces,
Moments are automatically balanced



Applied for



When all the masses are rotating
in same plane.

Dynamic Balancing



Here we balance both
forces as well as moment



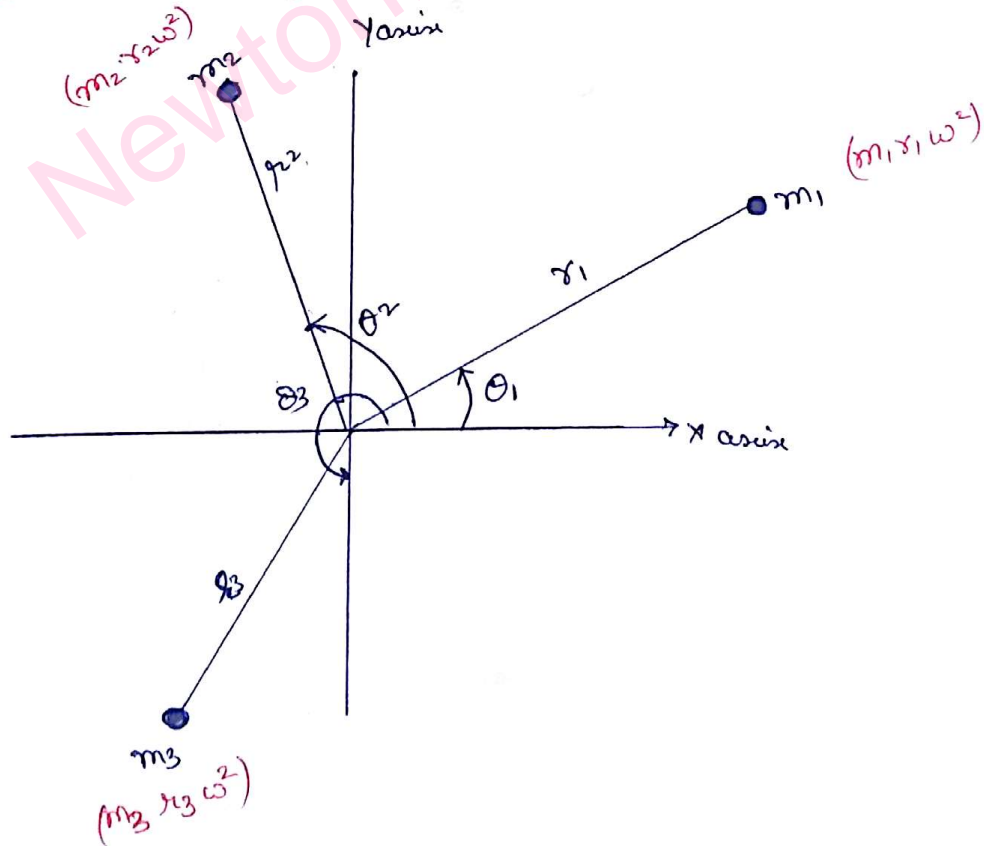
Applied for



When all the masses are
not rotating in same plane

Static Balancing of Rotating Masses:

All the masses are rotating in a same plane.



After Balancing

$$\sum \vec{F} = 0$$

$$F_x = 0 ; F_y = 0$$

$$F_x = 0$$

$$(m_1 r_1 \omega^2) \cos \theta_1 + (m_2 r_2 \omega^2) \cos \theta_2 + (m_3 r_3 \omega^2) \cos \theta_3 + (m_B r_B \omega^2) \cos \theta_B = 0$$

as Balancing doesn't depend on speed.

$$(m_B r_B) \cos \theta_B = - \sum m r \cos \theta \quad \text{--- ①}$$

$$(m_B r_B) \sin \theta_B = - \sum m r \sin \theta \quad \text{--- ②}$$

Squaring ① and ② and then adding them,

$$m_B r_B = \sqrt{(-\sum m r \cos \theta)^2 + (-\sum m r \sin \theta)^2} \quad **$$

for angle where mass to be attached for balancing

$$\tan \theta_B = \frac{-\sum m r \sin \theta}{-\sum m r \cos \theta} \quad **$$

Note

The final sign of Numerator & denominator will decide the Quadrant of Balance mass.

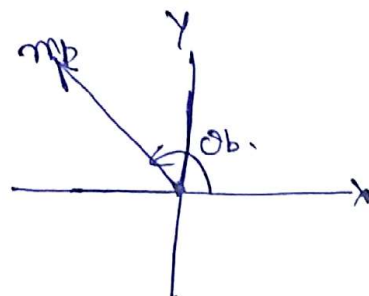
for example

$$\sum m r \cos \theta = +4$$

$$\sum m r \sin \theta = -5$$

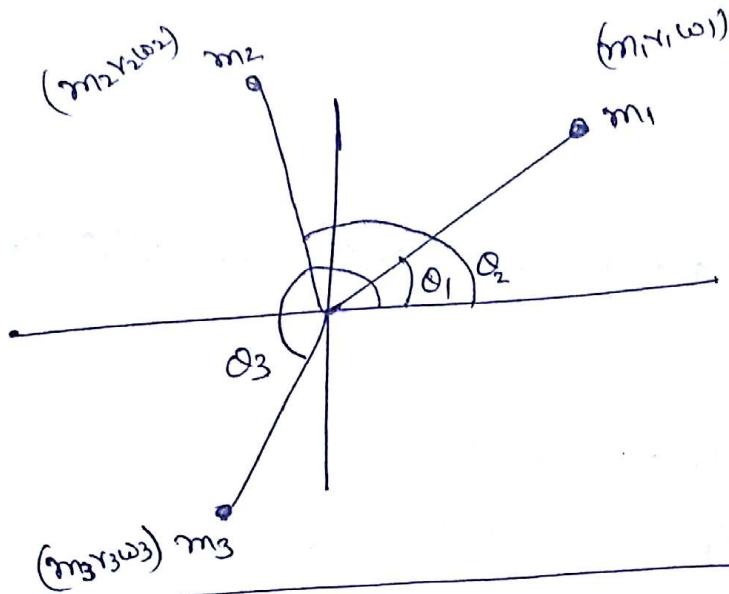
$$\tan \theta_B = \frac{-5}{+4} = -$$

$$\tan \theta_B = \frac{-(-5)}{-4} = \frac{5}{-4} = -\frac{5}{4}$$

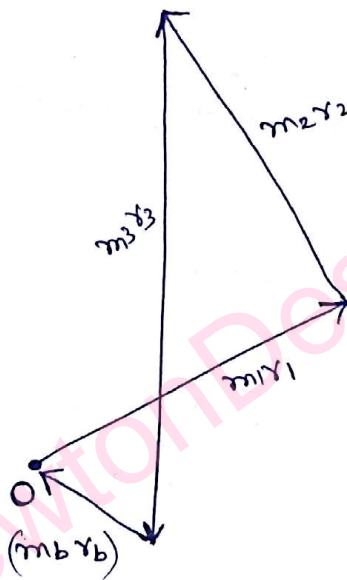


Graphical Method

$$\boxed{F/w^2}$$



m_1, r_1
 m_2, r_2
 m_3, r_3 } known
 $m_B, r_B \rightarrow$ unknown



Dynamic Balancing

(All the masses are not rotating in same Plane)

WorkBook Question 30 Pg. 40

$M_B = ?$

$M_D = ?$

$\theta_B = ?$

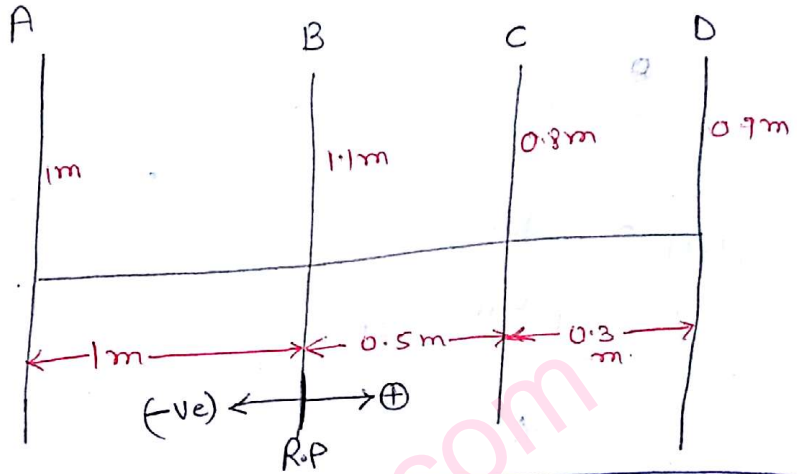
$\theta_D = ?$

$M_A = 5 \text{ kg}$

$M_B = ?$

$M_C = 3 \text{ kg}$

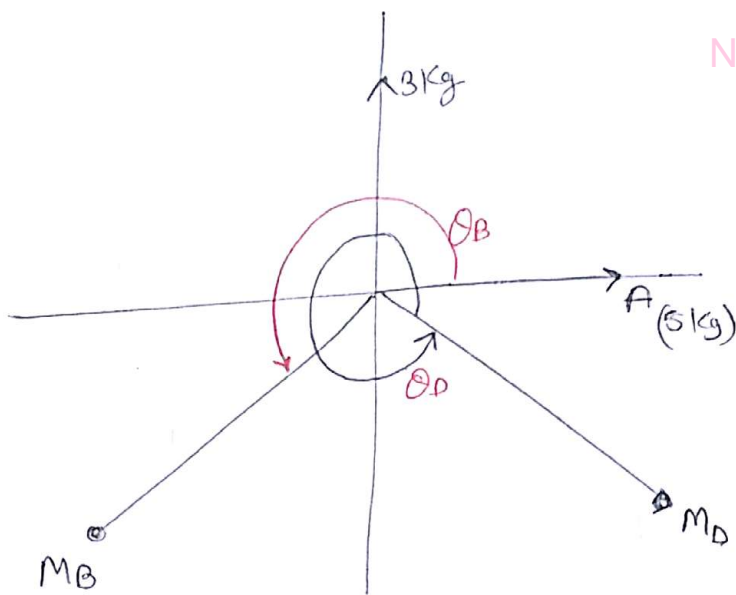
$M_D = ?$



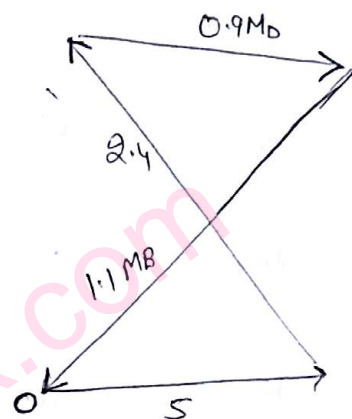
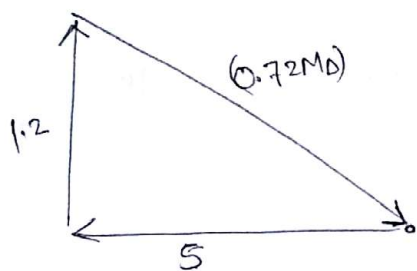
circ* $\text{Force} = mR\omega^2$
 $\frac{F}{\omega^2} = mR$

Plane	m	r	Force $\cdot \omega^2$ $m \cdot r$	Distance from (R.O.P.) (L)	Moment $m \cdot R \cdot L$
A	5	1	5	-1	-5
B	M_B	1.1	$1.1 M_B$	0	0
C	3	0.8	2.4	0.5	1.2
D	M_D	0.9	$0.9 M_D$	0.8	$0.72 M_D$

Configuration Dia



Moment Polygon



For Moment Polygon Value:

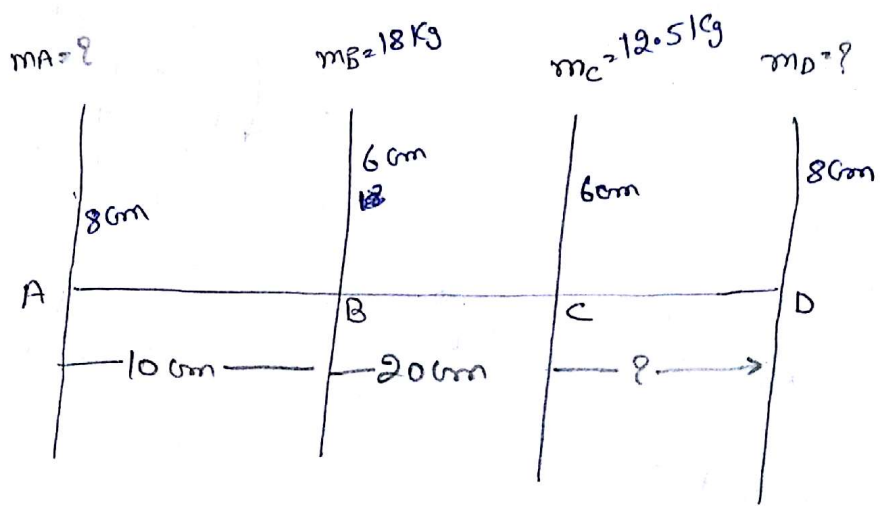
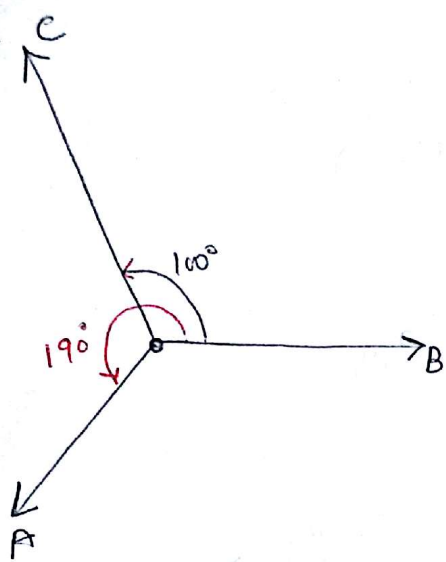
- ① Reference Plane Select करना है दिⁿ Show hai.
 - ② Main target if one value become zero.
- $(m \cdot r) \cdot l$
 ↓
 Distance from Reference Plane,
- जो Value -ve आरगी उसे out नाना है लि^ख

and +ve as usual

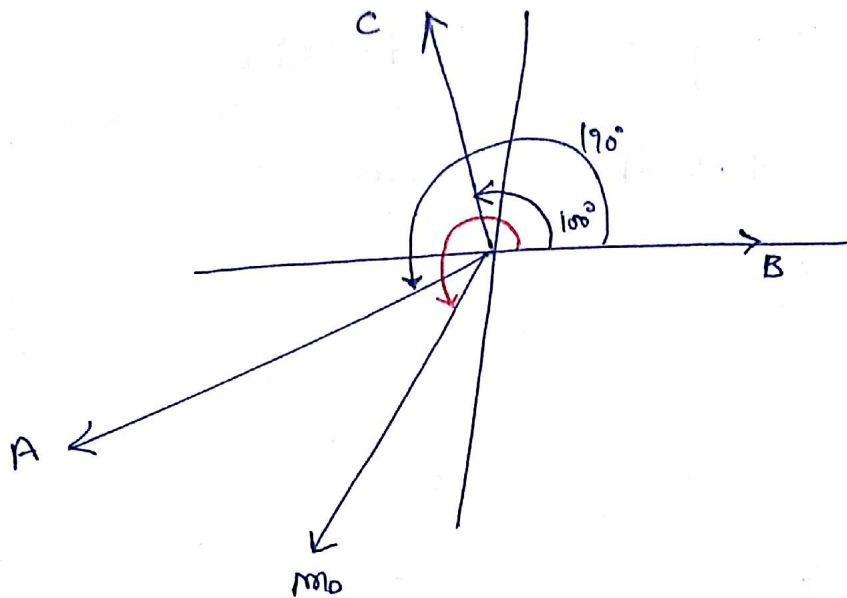


$\frac{\text{Force}}{w} \equiv m \cdot R$ Always ~~big~~ out. never be in.

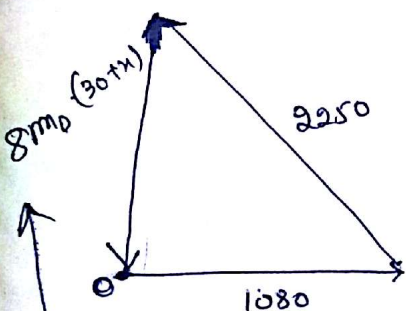
Q.3) T5) []



Plane	m	x	m · x	Distance from (R.P) (x)	m · RL
A (R.P)	m_A	8	$8 m_A$	0	0
B	18	6	108	10	1080
C	12.5	6	75	30	2250
D	m_D	8	$8 m_D$	$30+x$	$8 m_D (30+x)$

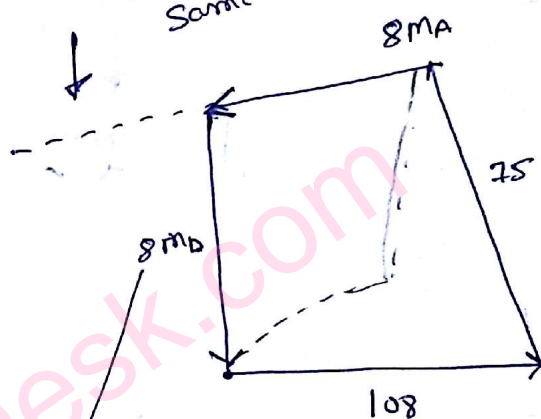


Moment Polygon



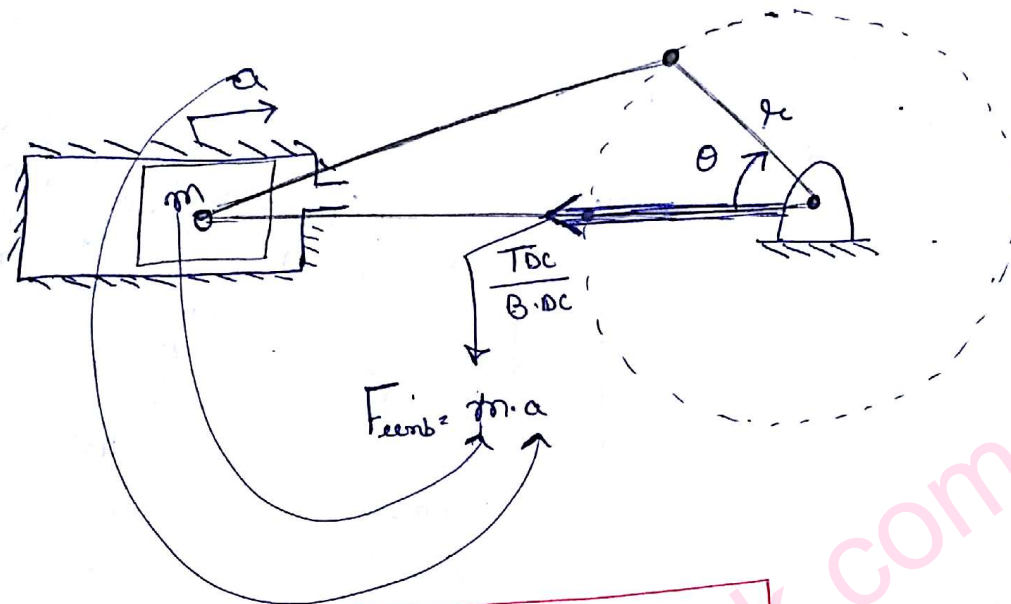
de-scale $8M_b$

यहाँ A की direction पता length नहीं है
 Same as D सो जिस line पर कोटिंग
 की होगा एंगल Plane.



Balancing of Reciprocating Masses :-

$m \rightarrow$ mass of Reciprocating ~~mass~~ Parts



$$F_{\text{unbalanced}} = m r \omega^2 \left\{ \cos \theta + \frac{\cos 2\theta}{n} \right\}$$

Always Constant in dirⁿ (line) but every moment changing in magnitude.

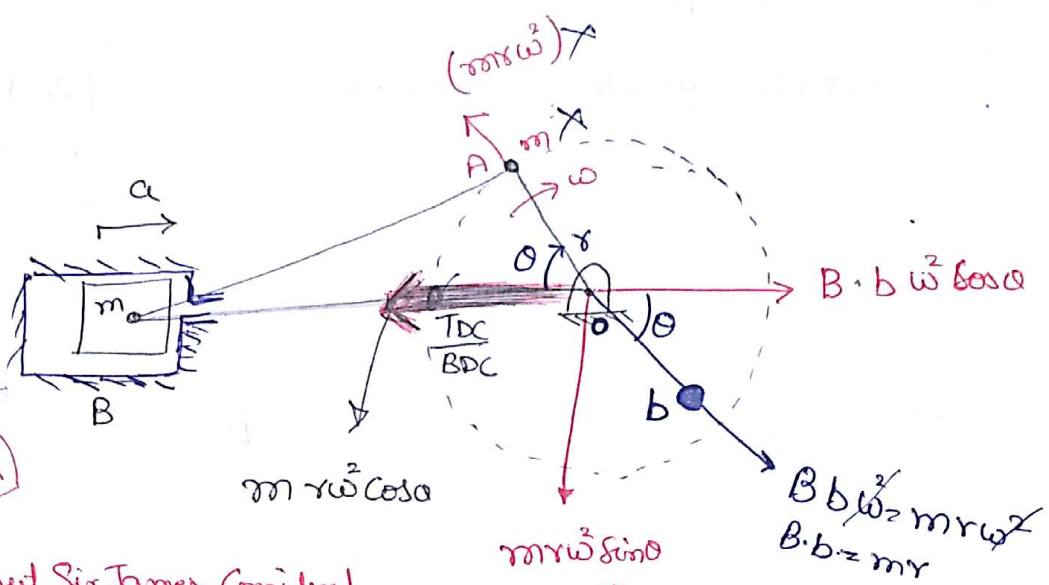
$$F_{\text{unb}} = \underbrace{(m r \omega^2 \cos \theta)}_{F_{\text{unb}} (\text{Primary})} + \underbrace{\left(m r \omega^2 \frac{\cos 2\theta}{n} \right)}_{F_{\text{unb}} (\text{Secondary})}$$

Primary Balancing of Reciprocating Masses :

Sir James Watt & Team

Artificial thinking

①



Vibration due to line of Reciprocation

But Sir James considered it as rotational, he succed by New developed ↑

They failed in the Balancing of Reciprocating masses. We will not Balance the Reciprocating masses completely, we will balance them Partilly

Partial Balancing of Reciprocating masses (Primary) (By Mr. Allen Casley)

Let $c \rightarrow$ fraction of Reciprocating mass to be balanced. $0 < c < 1$

$$B \cdot b = c m r < m \cdot r$$

After Partial Balancing

$$F_{unb} (\text{Along } \cos) = m r \omega^2 \cos \theta - B \cdot b \omega^2 \cos \theta = (1 - c) m r \omega^2 \cos \theta$$

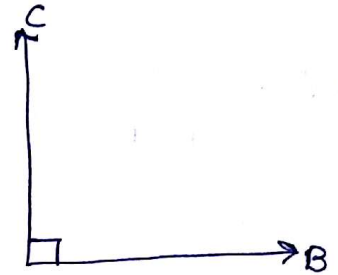
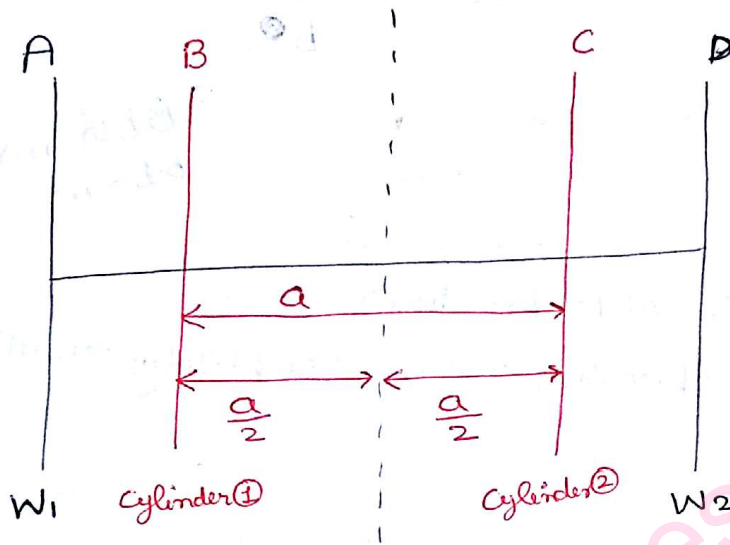
$$F_{unb} (\perp \text{ to } \cos) = B b \omega^2 \sin \theta = c m r \omega^2 \sin \theta$$

first use in locomotive
 → Vibration are there but intensity is less

Two Cylinder Locomotive :

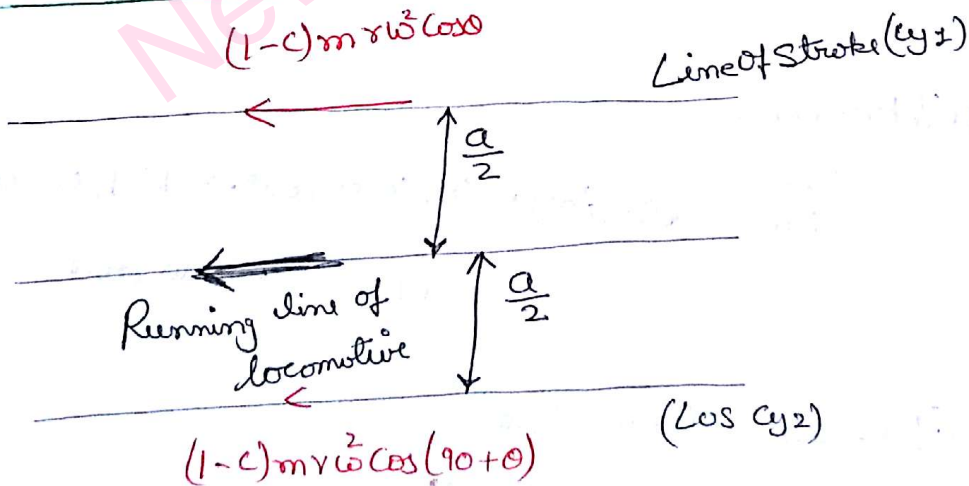
- Inside Cylinder Locomotive ✓
- Outside Cylinder Locomotive ✗

"First time to start the engine both of the cranks were kept 90° to each other".



Effect of Partial Balancing on two cylinder locomotive :-

i) Variation in Tractive forces :- Peelly



$$\text{Tractive force} = (1-c) m r \omega^2 \sin \theta + (1-c) m r \omega^2 \cos(90+\theta)$$

$$= (1-c) m r \omega^2 (\cos \theta - \sin \theta)$$

For Maximum differentiate it,

$$\frac{d}{d\theta} (\cos \theta - \sin \theta) = 0$$

$$\boxed{\tan \theta = -1}$$

$$(\text{at } \theta = 135^\circ, 315^\circ)$$

ii) Variation in Swaying Couple → अर्थ

$$\begin{aligned} \text{Swaying Couple} &= (1-c) m r \omega^2 \cos \theta \frac{a}{2} - (1-c) m r \omega^2 \sin(90+\theta) \frac{a}{2} \\ &= \frac{a}{2} (1-c) m r \omega^2 (\cos \theta + \sin \theta) \end{aligned}$$

For Max; differentiate it,

$$\frac{d}{d\theta} (\cos \theta + \sin \theta) = 0$$

$$\boxed{\tan \theta = 1}$$

$$\theta = 45^\circ, 225^\circ$$

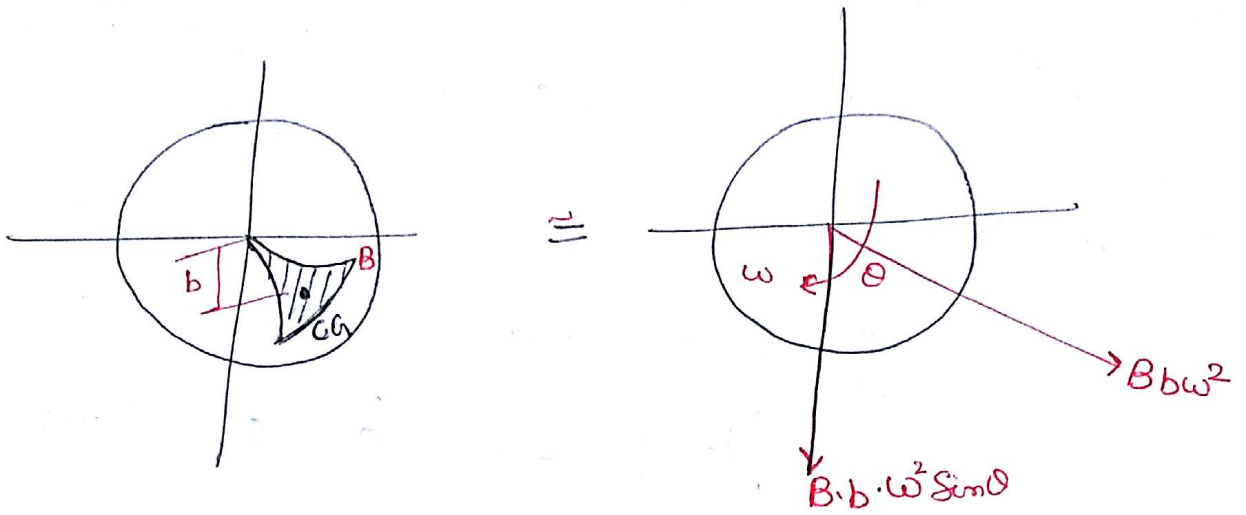
$$\text{Variation in Swaying Couple} = \pm \frac{a}{\sqrt{2}} (1-c) m r \omega^2$$

iii) Hammer Blow

In Reality the balance masses cannot be attached reverse to the crank.

In reality balance masses are attached on the wheels.

$$\boxed{B \cdot b \neq C \cdot m r}^*$$



$$(B b \omega^2 \sin \theta)_{\max} = B b \omega^2$$

⇓
Hammer blow

$(B b \omega^2) \leq \text{Per wheel static load}$

→ Speed limited

10% of total Power of engine is used

Hammer Blow = $(B) b \omega^2$. (per wheel)

only that Balance mass will be responsible for Hammer Blow which is required for Reci. balancing.

Prob
(31)

Cy. Planes gap = 0.7 m

$r = 0.3 \text{ m}$

Crank are at 90°

Rot. Mass/cy. = 160 kg

Reci. mass/cy. = 180 kg

Balance \rightarrow Rot. + $\frac{2}{3}$ Reci.

$b = 0.8 \text{ m}$

wheels planes gap = 1.5 m.

$N = 300 \text{ r.p.m.}$

Balance Balance

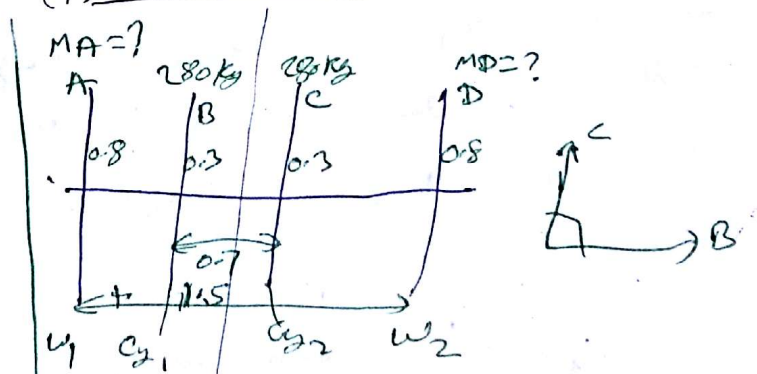
Rot. + $\frac{2}{3}$ Reci.

$= 160 \text{ kg} + \frac{2}{3} \times 180 \text{ kg}$

$= 160 \text{ kg} + 120 \text{ kg}$

$= 280 \text{ kg/cy.}$

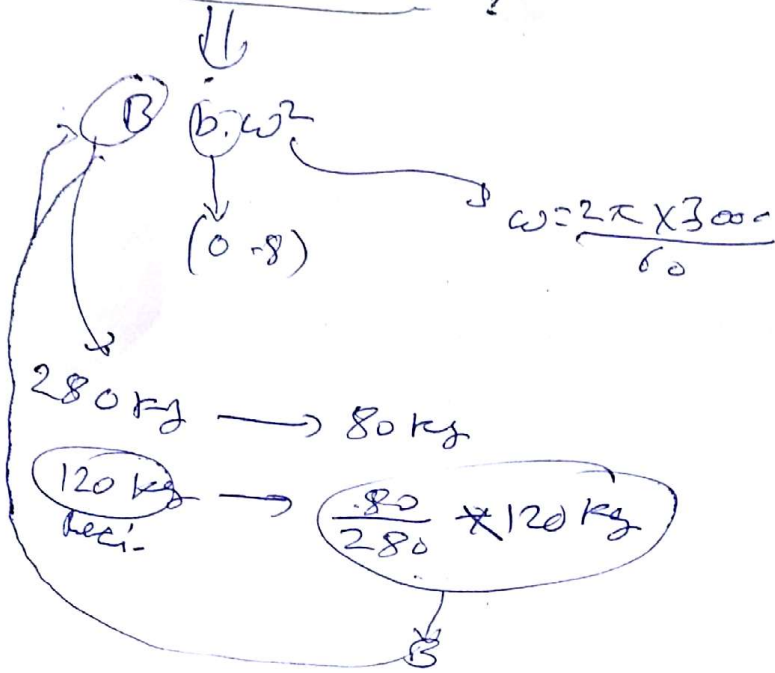
(i) BALANCING



$M_A = ?$ Same
 $M_D = ?$
 $Q_A = ?$ Different
 $Q_D = ?$

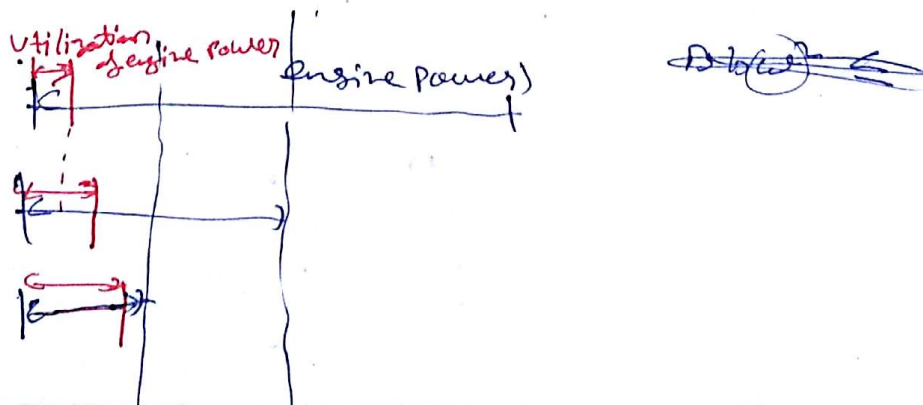
for exam
 $M_A = M_D$
 $= 80 \text{ kg}$

11) Hammer Blow:-?

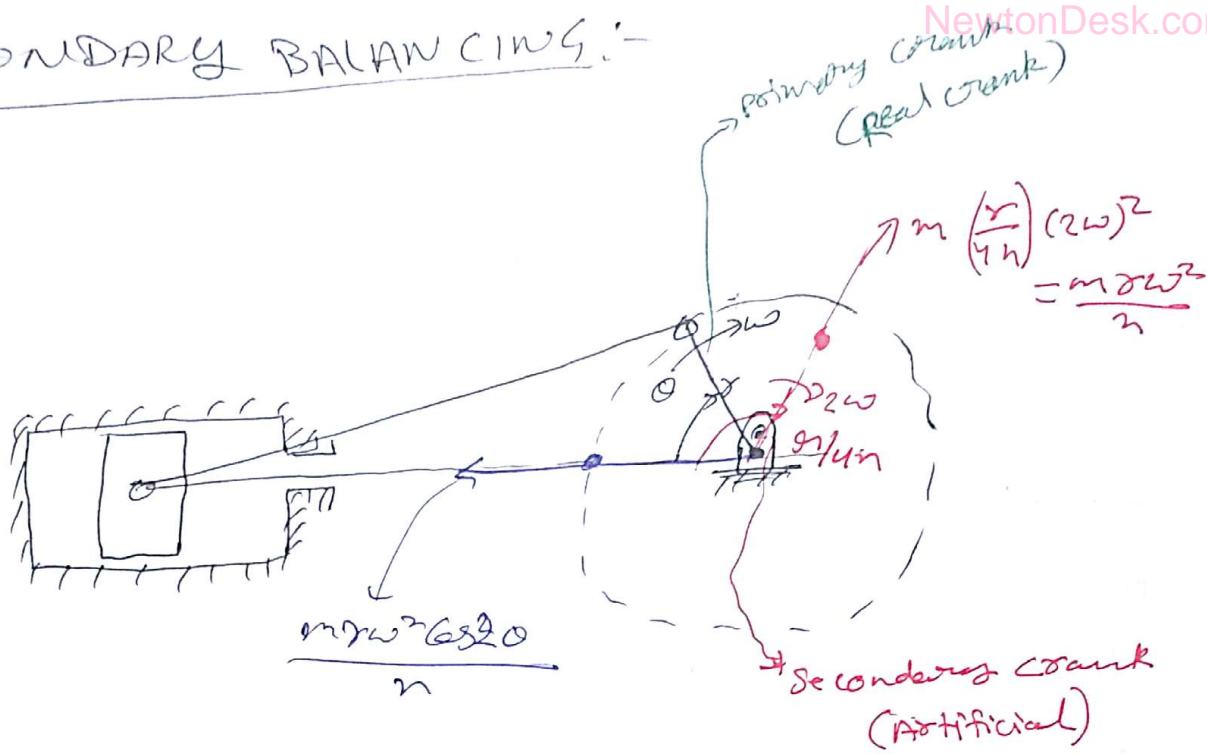


INTRODUCTION OF COUPLING ROD IN TWO CY. LOCOMOTIVES

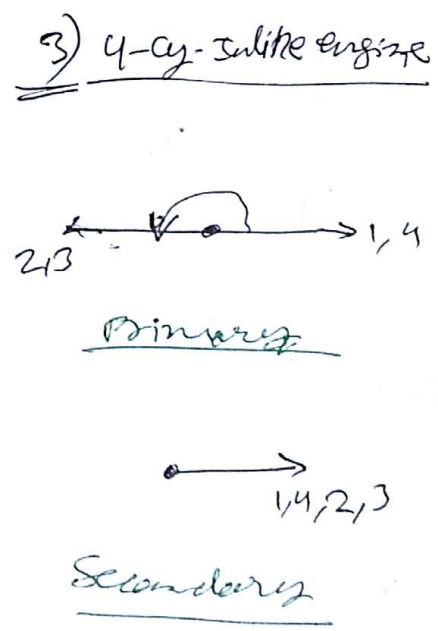
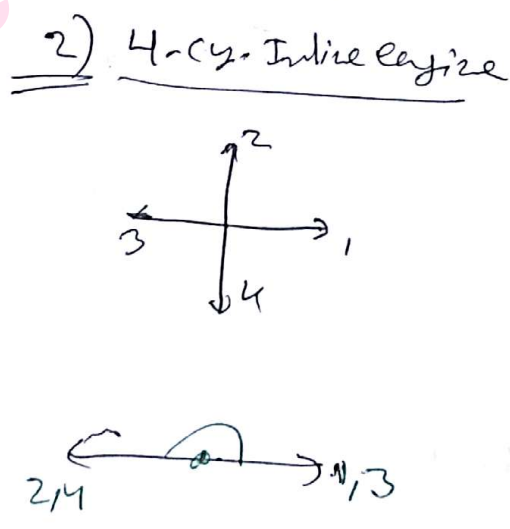
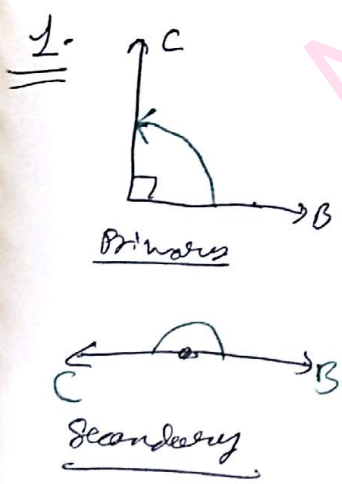
Coupling Rod was basically introduced to decrease the amount of Hammer blow by doing the BALANCE mass requirement for the reciprocating. Balancing through splitting the reciprocating balancing b/w the driving & trailing wheels. This was the development from passenger locomotives to the Express locomotives (in which 2-2 wheels were coupled) to the Superfast locomotives (in which 3-3 wheels were coupled).



SECONDARY BALANCING :-



Primary	Secondary
$m r w^2 \cos \theta$	$\frac{m r w^2 \cos 2\theta}{n}$
	$= m \left(\frac{r}{4n} \right) (2w)^2 \cos 2\theta$

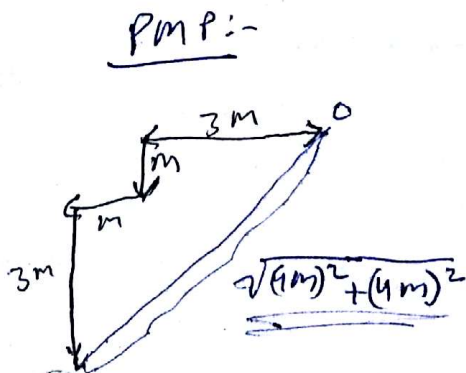
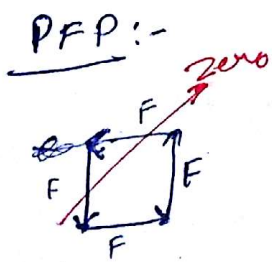
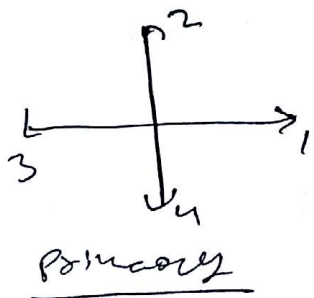
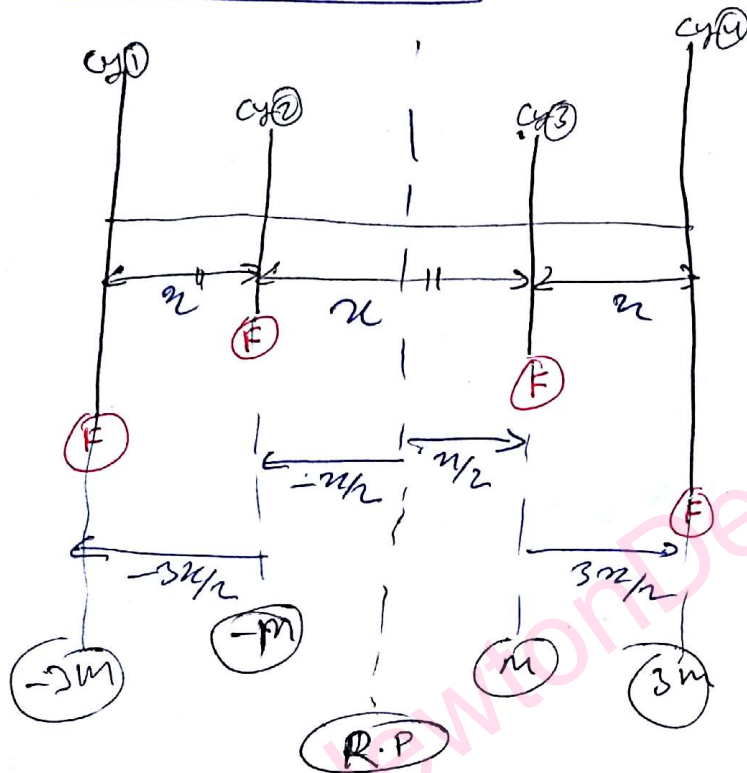


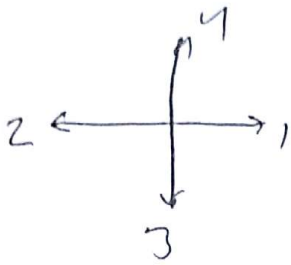
ROLL OF THE FIRING ORDER IN

MULTICYLINDER INLINE ENG. IN BALANCING

- PFP → Primary force Polygon
- PMP → " Moment " "
- SFP → Secondary Force " "
- SMP → " moment " "

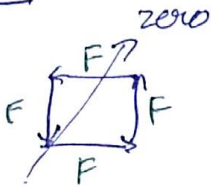
4-cy. Inline Engine :-



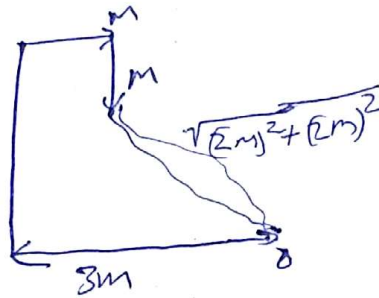


Primary

PFP



PMP :-

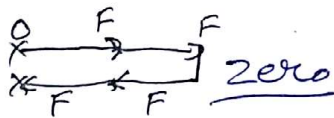


The real firing order of 4-cy. Inline engine in practical is

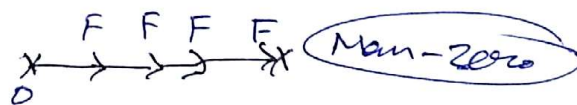
Best in 4-cyleng.



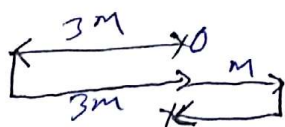
PFP



SFP :-

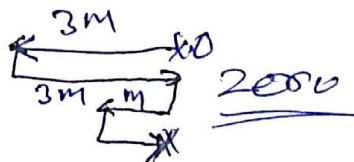


PMP :-



zero

SMP :-



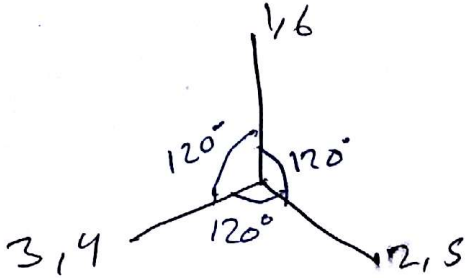
NOTE

4-Cy. Inline engine

↳ Most Completely Balanced.

6-Cy. Inline engine

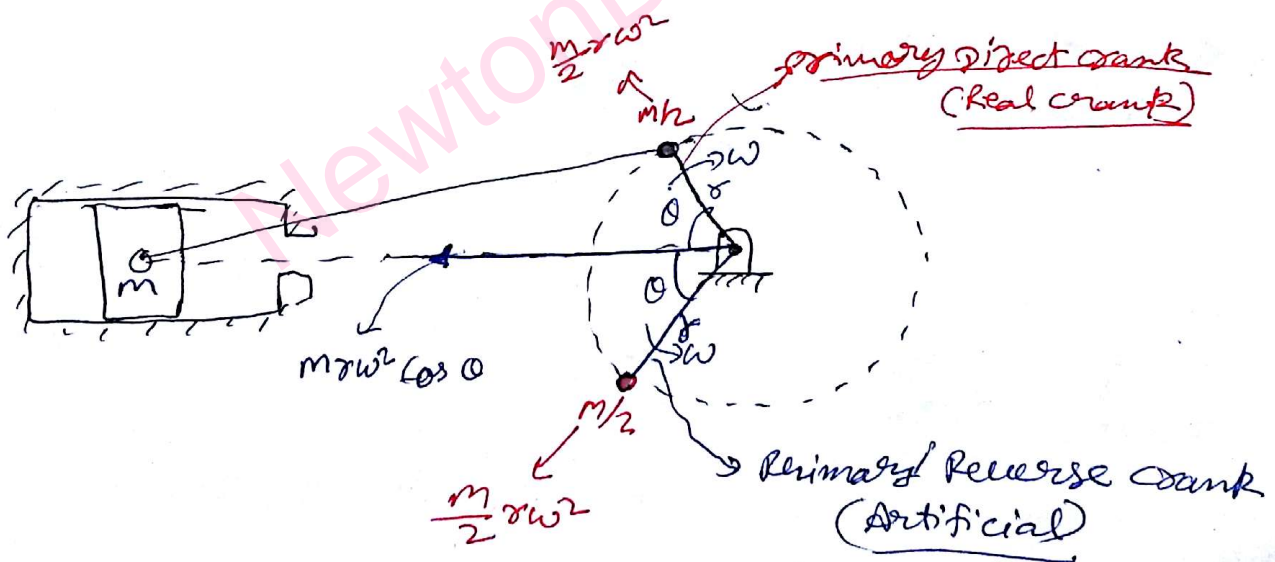
Completely Balanced



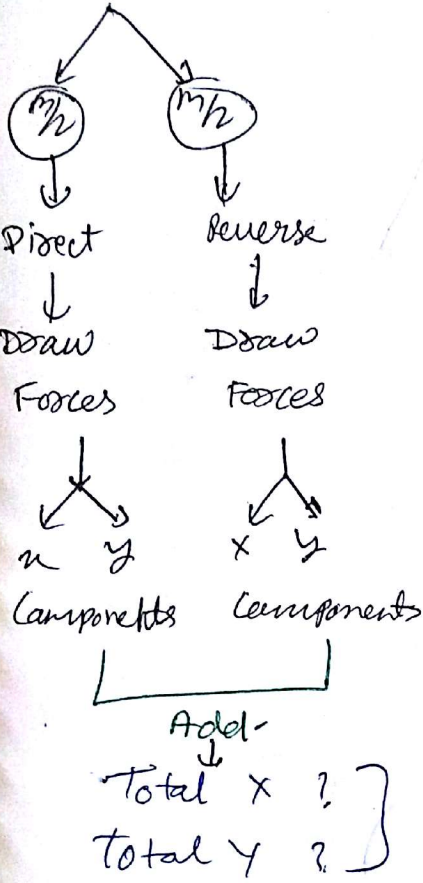
← This F. order used in Highly costly cars.
Highly (BMW, Ferrari)

DIRECT & REVERSE CRANK METHOD:-

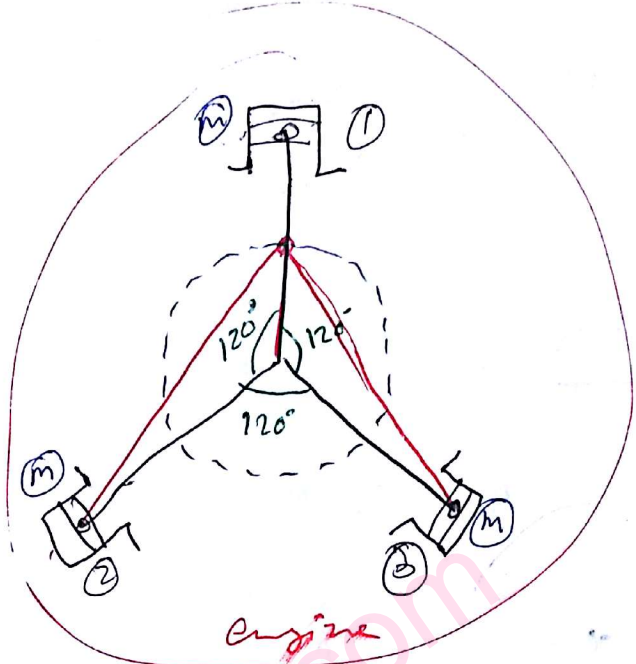
" This method is basically applied in multi-cylinder radial engine to get the magnitude & direction of primary & secondary unbalanced forces!"



(M) Reciprocating mass

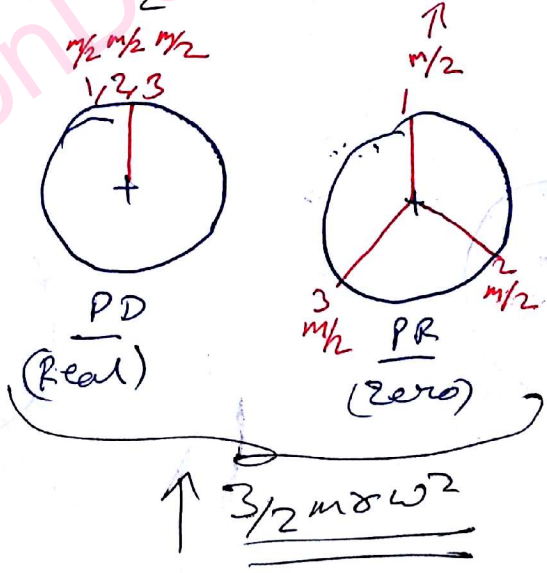


Three Cr. Radial Engine
in radial there is only one crank.

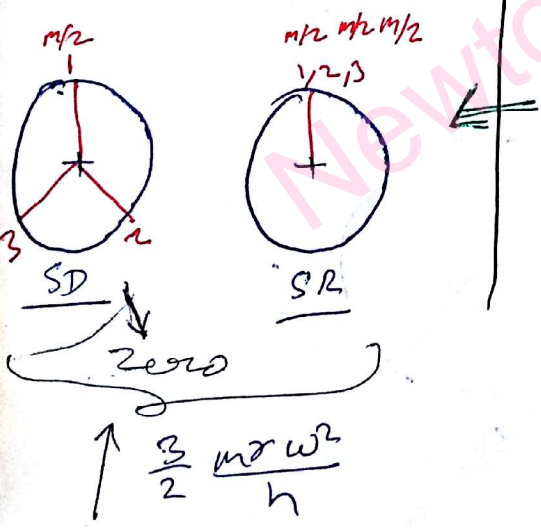


$F_{Primary} = ?$

$3 \times \frac{m}{2} r \omega^2$



$F_{Secondary} = ?$

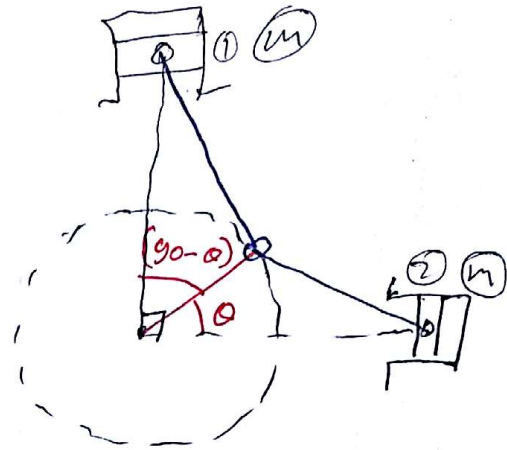


Prob

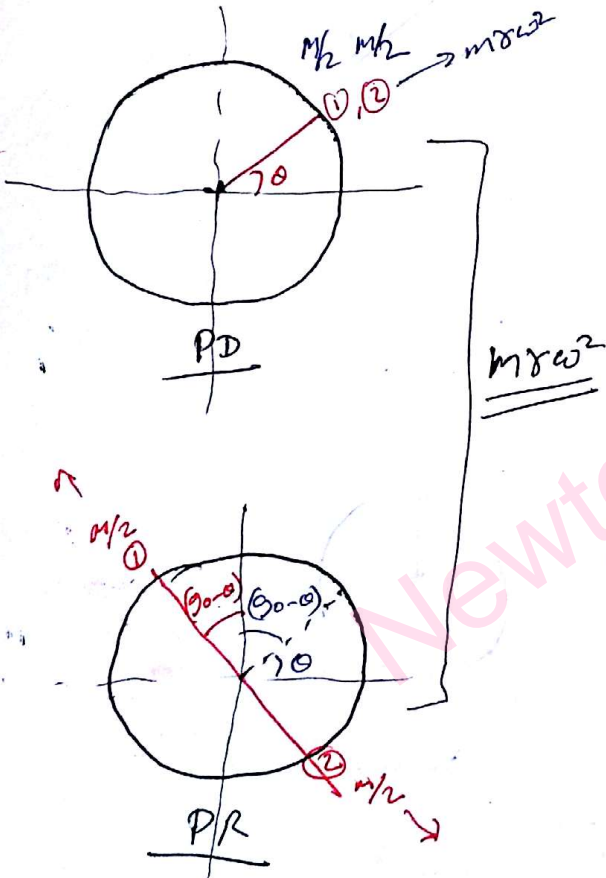
90° - V - engine

$F_{primary}$ → max ?
 → min ?

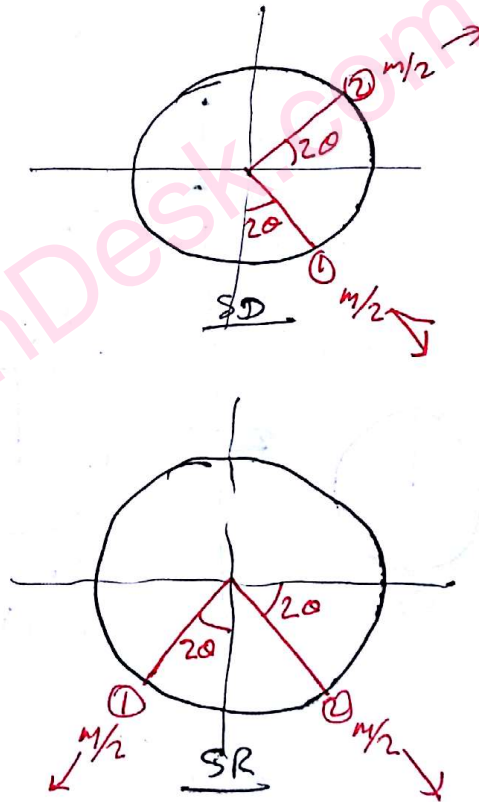
 $F_{secondary}$ → max ?
 → min ?



Primary



Secondary

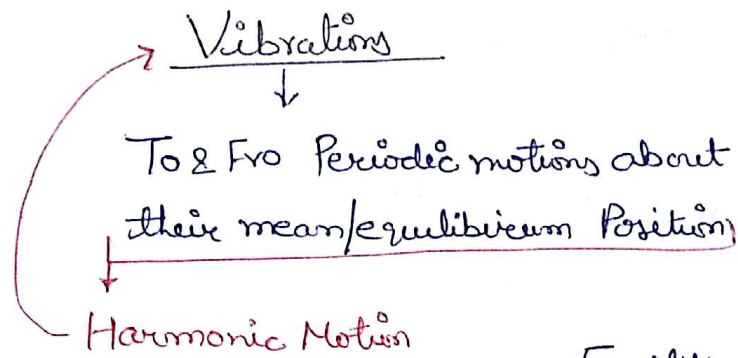


$$F_x = \frac{m}{2} \frac{r \omega^2}{n} [\cos^2 \theta + \sin^2 2\theta + \cos^2 2\theta - \sin^2 \theta]$$

$$\rightarrow F_x = \frac{m r \omega^2}{n} \cos 2\theta$$

$$F_y = \frac{m}{2} \frac{r \omega^2}{n} [\sin 2\theta - \cos 2\theta - \sin 2\theta - \cos 2\theta]$$

$$\rightarrow F_y = -\frac{m r \omega^2}{n} \cos 2\theta \quad \left| \quad F_{secondary} = \sqrt{2} \frac{m r \omega^2}{n} \cos 2\theta \rightarrow \text{max}$$

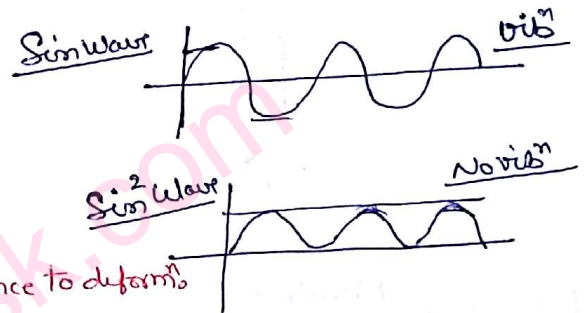


Equilibrium Position } having
Mean Position } Force (Net)
Zero Position } & Net Moment
Zero.

→ Vibrations are introduced b/c,
of some initial disturbances.

Any Vibration System

- i) K.E Storing device (mass)
- ii) P.E Storing device (Stiffness) → Resistance to deform^s
- iii) Energy loss due to kinetic friction $\neq 0$
- iv) Unbalanced forces $F_{un} \neq 0$

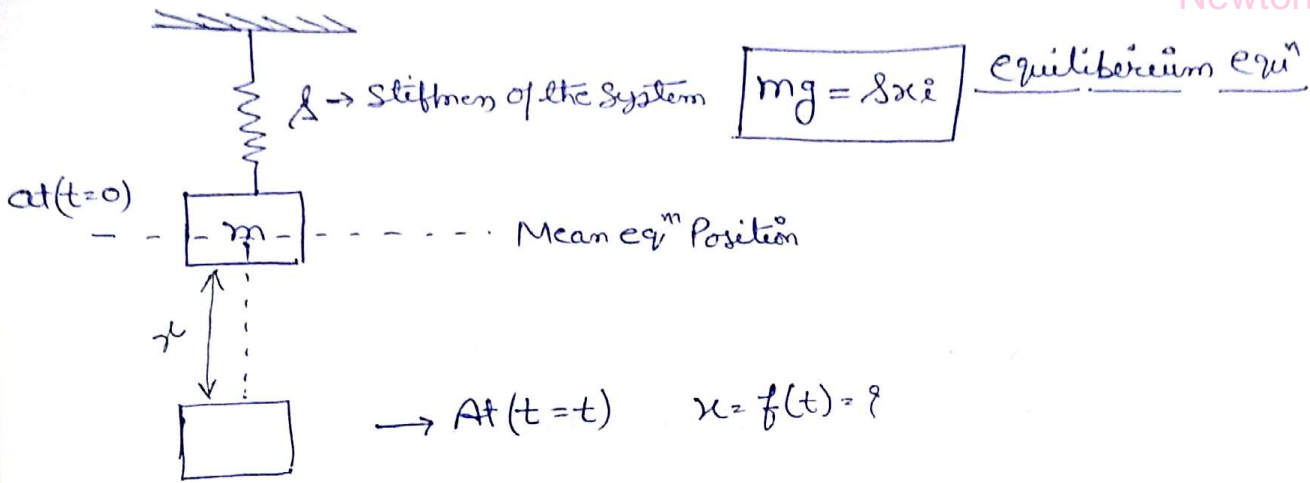


i, ii, iii are Basic Cause of Non-Running vibrating system.

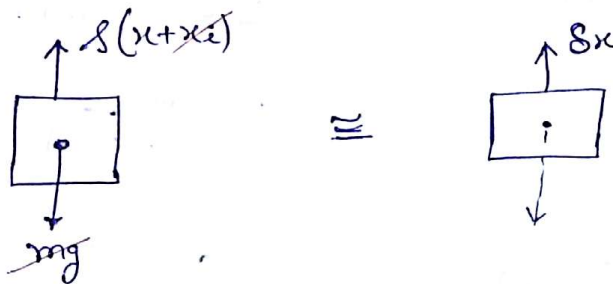
i, ii, iii including iv are Basic Cause of Running vibrating system.

Natural Vibrations :- By Sir Galileo →

"The vibrations in which there is no kinetic friction at all, as well as there is no external force, after the initial Release of the System, are known as Natural vibrations".



At $t=t$ System Free body diagram



By using Newton's Second Law:

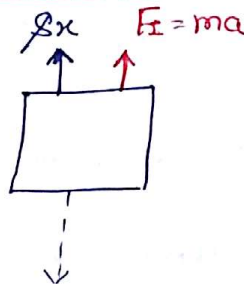
$$0 - \Delta x = ma$$

$ma + \Delta x = 0$

**

D'Alembert Principle: Father of inertia force.

System (F.BD)
At $(t=t)$



$ma + \Delta x = 0$

**

$$a + \left(\frac{\Delta}{m}\right)x = 0$$

$\ddot{x} + \frac{\Delta}{m}x = 0$

**

Eqⁿ of Natural System

$$\ddot{x} + \frac{s}{m} x = 0$$

Solution of above eqnⁿ is

$$x = R \sin \left(\sqrt{\frac{s}{m}} t + \phi \right)$$

Where R & ϕ are Constant

Amplitude
↓
Constant

vibration with frequency

$$\omega_n = \sqrt{\frac{s}{m}} \text{ rad/s} \quad \text{Angular frequency}$$

$$T_n = \frac{2\pi}{\omega_n} \text{ sec} \quad \text{time period}$$

$$f_n = \frac{\omega_n}{2\pi} \text{ Hz} \quad \text{linear frequency}$$

→ R, ϕ are found by Initial condition i.e. at $t=0$

i) At $t=0$ $\begin{cases} x = x_0 \\ \dot{x} = 0 \end{cases}$

ii) At $t=0$ $\begin{cases} x = 0 \\ \dot{x} = v_0 \end{cases}$

iii) At $t=0$ $\begin{cases} x = x_0 \\ \dot{x} = v_0 \end{cases}$

finally, equation of Natural vibration

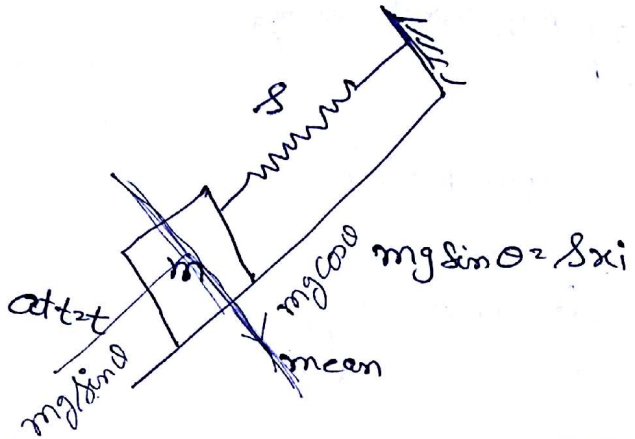
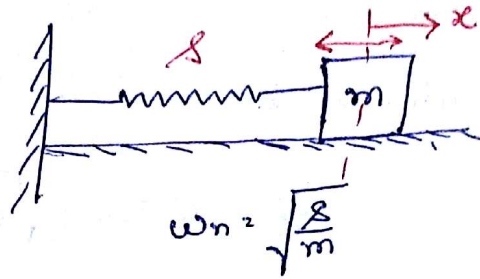
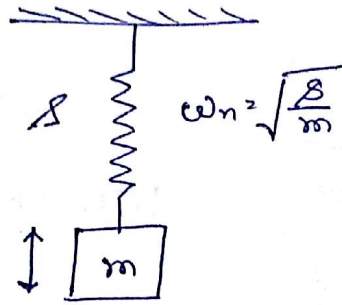
$$\ddot{x} + \omega_n^2 x = 0$$

Q2) The Natural vibration system eqnⁿ is Relate with this

$$5\ddot{x} + 3x = 0$$

$$\ddot{x} + \frac{3}{5} x = 0$$

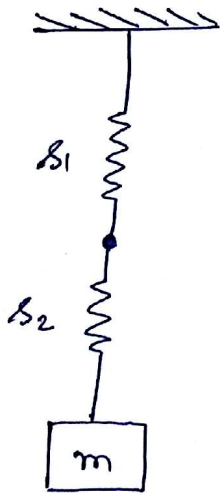
$$\omega_n = \sqrt{\frac{3}{5}} \text{ rad/sec} \quad \underline{An}$$



$S \propto \text{Area}$
 $S \propto \frac{1}{\text{Length}}$

Combinations of Springs

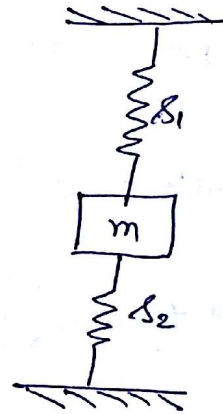
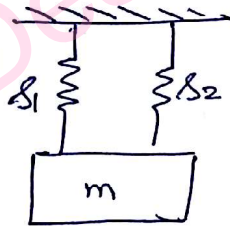
Series



$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2} \Rightarrow S = ?$$

$$\omega_n = \sqrt{\frac{S}{m}}$$

Parallel

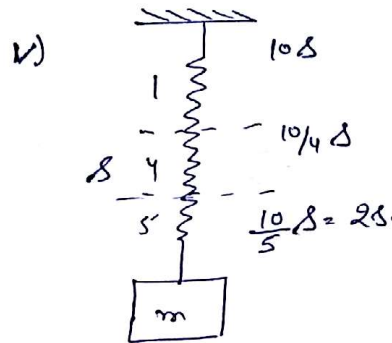
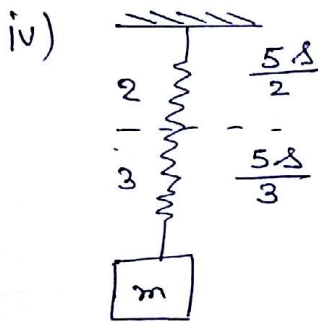
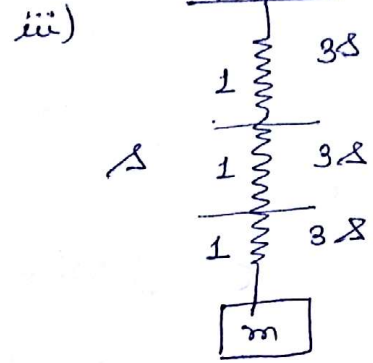
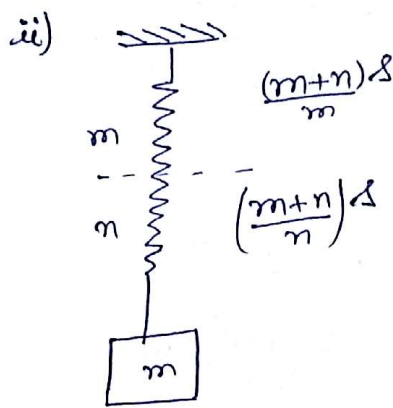
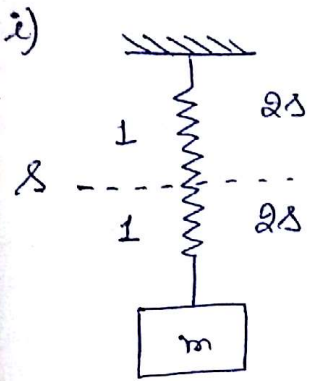


$$S = S_1 + S_2$$

$$\omega_n = \sqrt{\frac{S}{m}}$$

Both same

Cutting of Springs :



Energy Method :

In Natural vibration, Kinetic friction is 0. but static friction is there which have no effect.

Total energy (E) = Const.

$$\frac{dE}{dt} = 0$$

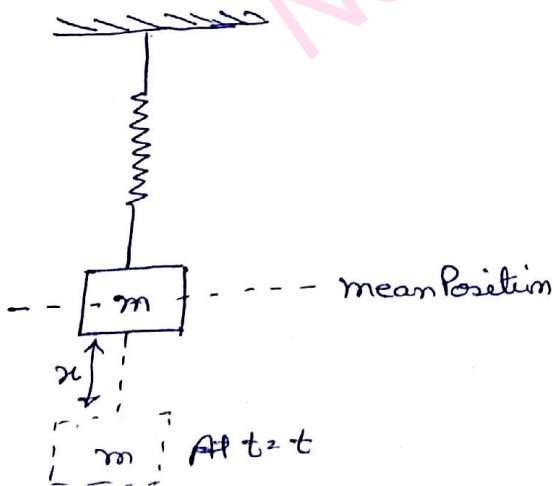
$$\frac{dE}{dt} = \frac{1}{2} m \cdot \frac{dv}{dt} + \frac{1}{2} s x \frac{dx}{dt} = 0$$

Acceleration

$$m\ddot{x} + sx = 0$$

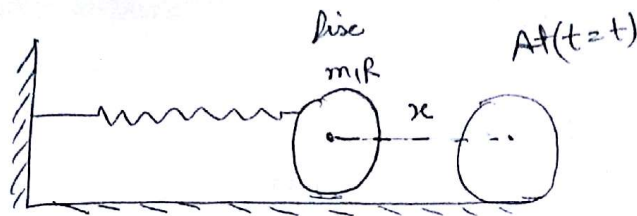
$$\ddot{x} + \frac{s}{m} x = 0$$

$$\omega_n = \sqrt{\frac{s}{m}}$$



$$\text{Energy} = \frac{1}{2} m v^2 + \frac{1}{2} s x^2$$

Problem 2M



At $t=t$

$$E = \frac{1}{2} \Delta x^2 + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \Delta x^2 + \frac{1}{2} m v^2 + \frac{1}{2} \left(\frac{m R^2}{2} \right) \omega^2$$

$$= \frac{1}{2} \Delta x^2 + \frac{1}{2} \left(\frac{3m}{2} \right) v^2$$

$$\omega_n = \sqrt{\frac{\Delta}{\frac{3m}{2}}} \Rightarrow \omega_n = \sqrt{\frac{2\Delta}{3m}}$$

Various Moment of Inertia:

Ring / Hollow cylinder $\Rightarrow I = mR^2$

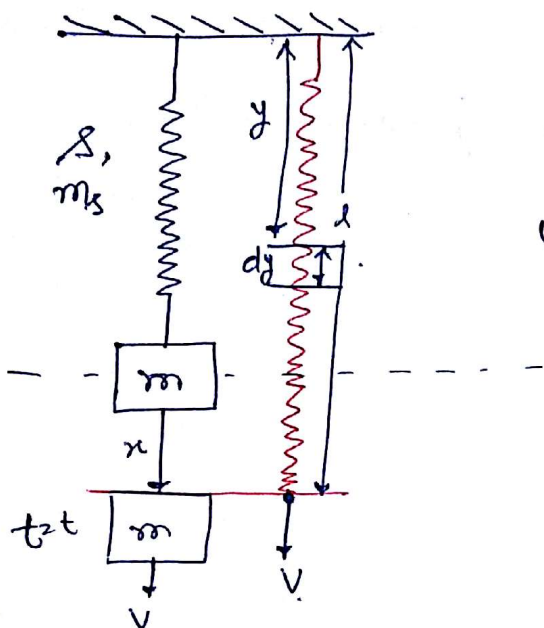
Disc / Solid cylinder $\Rightarrow I = \frac{mR^2}{2}$

Hollow sphere $\Rightarrow I = \frac{2}{3} mR^2$

Solid sphere $\Rightarrow I = \frac{2}{5} mR^2$

Spring Mass System

(Spring is also having mass)



$$u = \frac{v}{l} y$$

$$K.E = \int_0^l \frac{1}{2} \frac{m_s}{l} dy \cdot \left(\frac{v}{l} y\right)^2$$

$$= \frac{1}{2} \frac{m_s}{l} \frac{v^2}{l^2} \cdot \frac{l^3}{3}$$

$$K.E = \frac{1}{6} m_s v^2$$

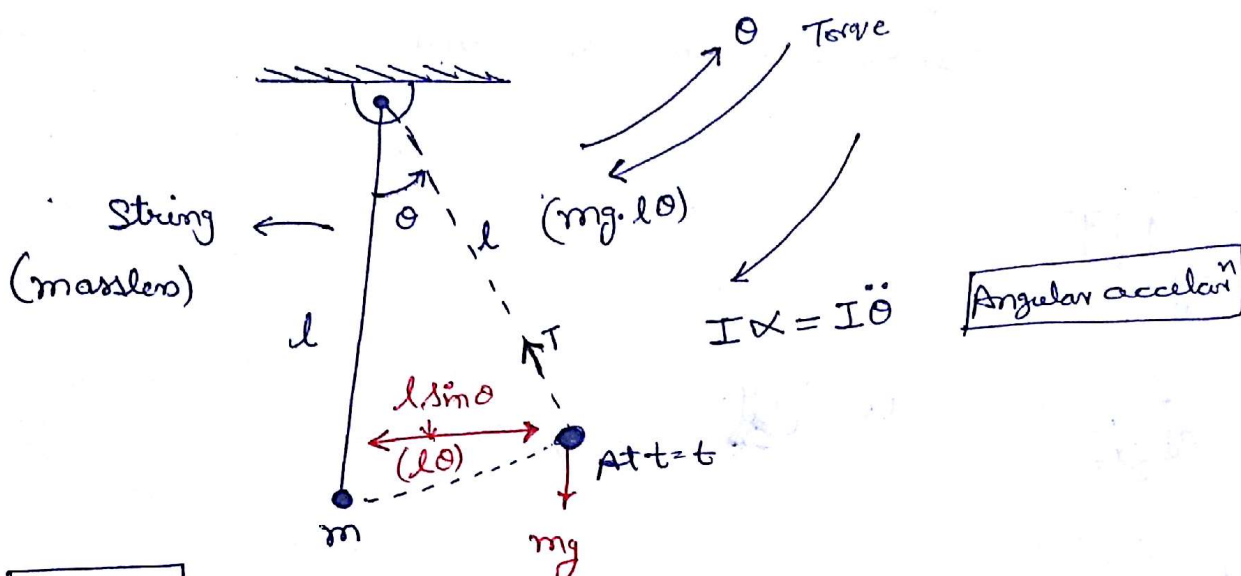
At $t+dt$

$$E = \frac{1}{2} S x^2 + \frac{1}{2} m v^2 + \frac{1}{6} m_s v^2$$

$$= \frac{1}{2} S x^2 + \frac{1}{2} \left(m + \frac{m_s}{3}\right) v^2$$

$$\omega_n = \sqrt{\frac{S}{m + \frac{m_s}{3}}}$$

Torque Method :- (for Small Oscillations)



$$I = ml^2$$

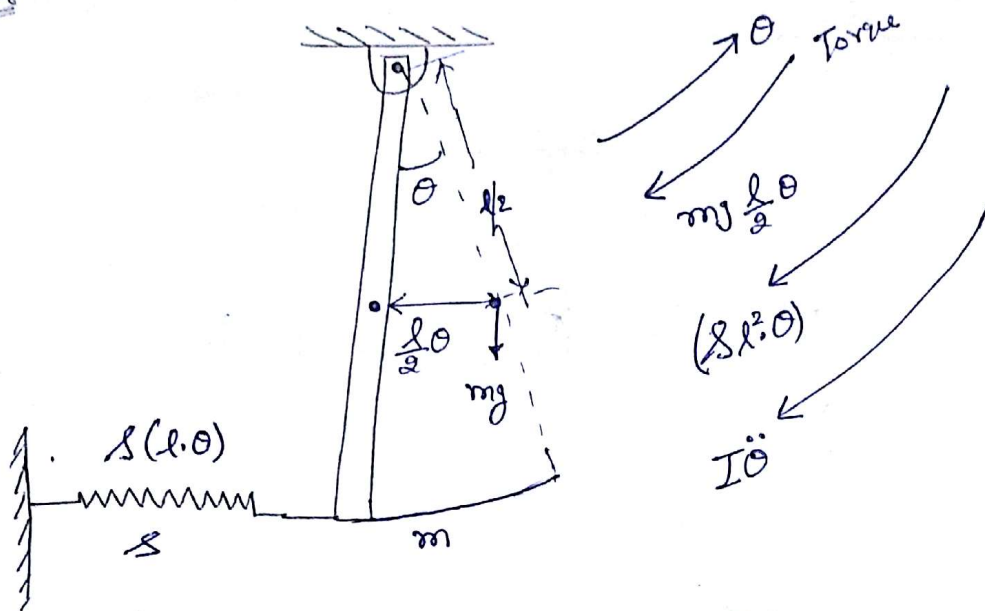
$$I \ddot{\theta} + (mg l) \theta = 0$$

$$\ddot{\theta} + \left(\frac{mg l}{I} \right) \theta = 0$$

$$\ddot{\theta} + \frac{mg l}{ml^2} \theta = 0$$

$$\ddot{\theta} + \frac{g}{l} \theta = 0$$

$$\omega_n = \sqrt{\frac{g}{l}} \text{ rad/sec}$$



$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

$$= \frac{ml^2}{12} + \frac{ml^2}{4}$$

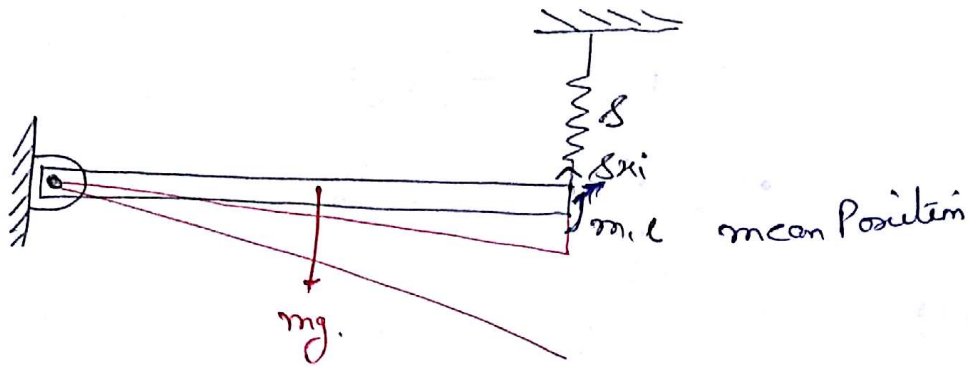
$$I = \frac{ml^2}{3}$$

$$I\ddot{\theta} + \frac{mgl}{2} + sl^2 = 0$$

$$\ddot{\theta} = \frac{\frac{mgl}{2} + sl^2}{ml^2/3} \theta = 0$$

$$\omega_n = \sqrt{\frac{\frac{mgl}{2} + sl^2}{\frac{ml^2}{3}}}$$

Note: Horizontal System



$$mg \frac{l}{2} = \delta x_i l$$

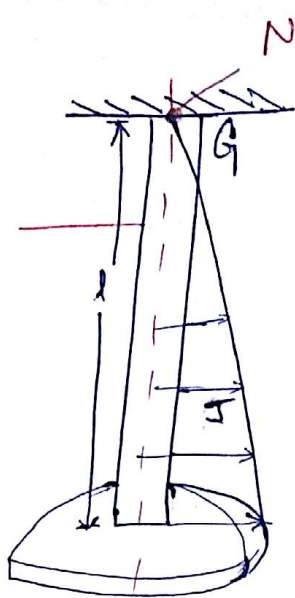
[mg torque is cancelled by δx_i torque
[mg torque will not be considered

$$\omega_m = \sqrt{\frac{\delta l^2}{\frac{ml^2}{3}}}$$

$$\omega_m = \sqrt{\frac{3\delta}{m}}$$

Torsional vibrations

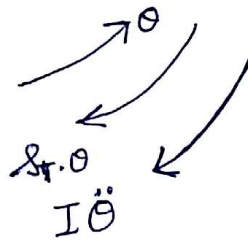
Rod
Masses



Torsional stiffness

$$\boxed{\mathcal{S}_T = \frac{GJ}{l}}^*$$

J → Polar moment of inertia (By Area)



$$I\ddot{\theta} + (\mathcal{S}_T)\theta = 0$$

$$\ddot{\theta} + \frac{\mathcal{S}_T}{I}\theta = 0$$

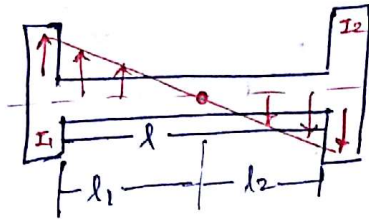
$$\boxed{\omega_n = \sqrt{\frac{\mathcal{S}_T}{I}}}$$

Note

If Rod (shaft) is having mass. (I_{shaft})

$$\boxed{\omega_n = \sqrt{\frac{\mathcal{S}_T}{I + \frac{I_{shaft}}{3}}}}$$

Two Rotor System :



* If Number of Rotors = n
No of Node will always
Always $(n-1)$

Node - where Torsional vibration
are zero

$$l_1 + l_2 = l \quad - (1)$$

$$\sqrt{\frac{\delta T_1}{I_1}} = \sqrt{\frac{\delta T_2}{I_2}}$$

$$\frac{\delta T_1}{I_1} = \frac{\delta T_2}{I_2}$$

$$\frac{GJ}{l_1 I_1} = \frac{GJ}{l_2 I_2} \Rightarrow \frac{l_1}{l_2} = \frac{I_2}{I_1} \quad - (2)$$

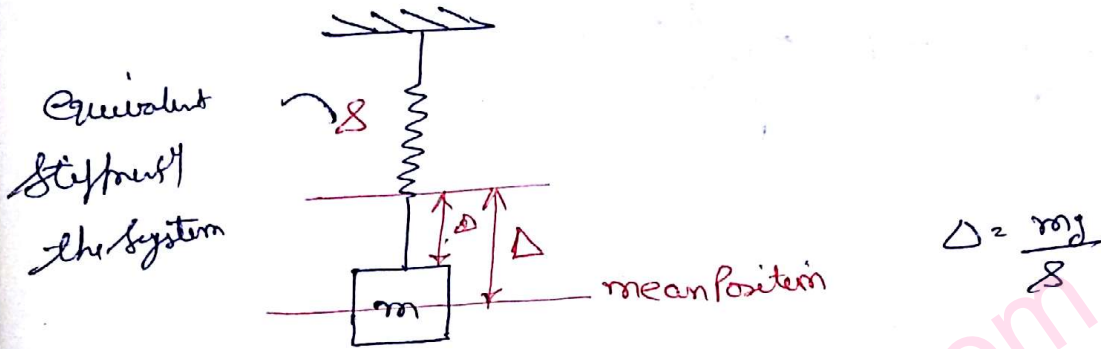
by (1) & (2) we can calculate l_1 & l_2

Rayleigh Method ∴

Method of Static Deflection of Mass Δ

$$\Delta \rightarrow \text{Mass}$$

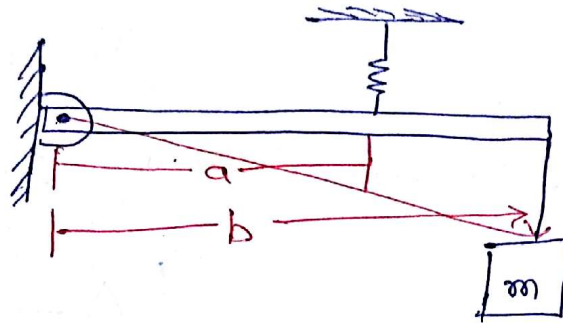
Basic Spring-mass system



$$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{\frac{mg}{s}}} = \sqrt{\frac{s}{m}} = \omega_n$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{s}{m}}$$

Pb 1)



$$F_2 \Delta_1 = \frac{mg(b)a}{s_1}$$

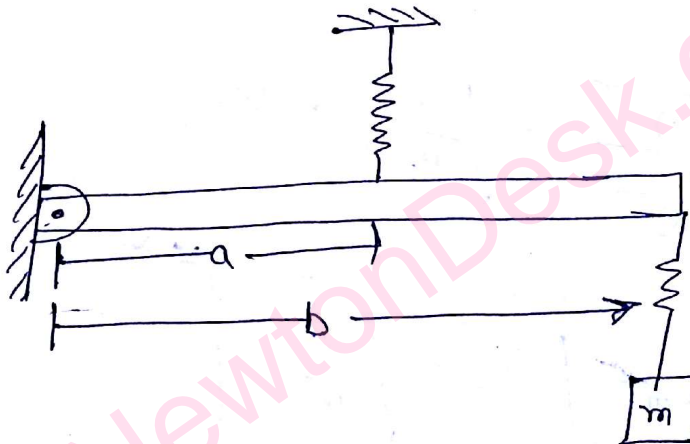
$$mg b = F \cdot a$$

$$\left[F = \frac{mg \cdot b}{a} \right]$$

$$\frac{\Delta}{b} = \frac{\Delta_1}{a} \Rightarrow \Delta = \Delta_1 \frac{b}{a} = \frac{mg}{s_1} \cdot \left(\frac{b}{a}\right)^2$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = ? = \sqrt{\frac{s}{m}} \rightarrow ??$$

Pb 2)

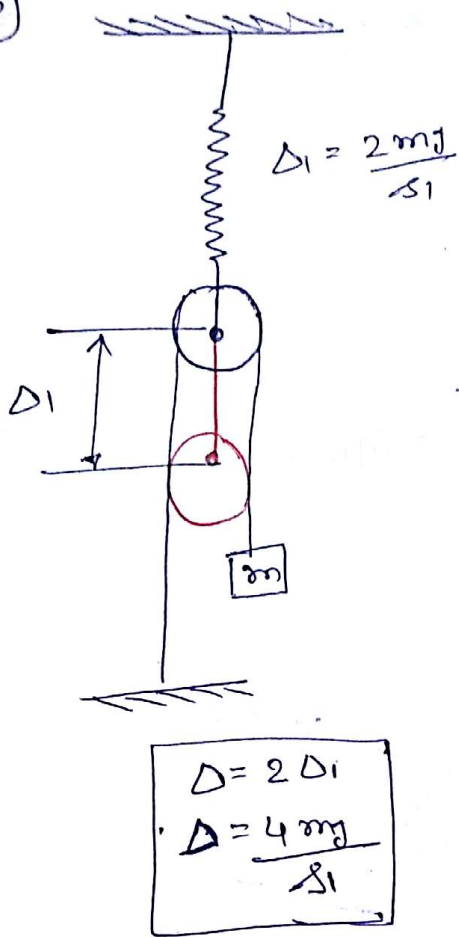


$$\Delta_2 = \frac{mg}{s_2}$$

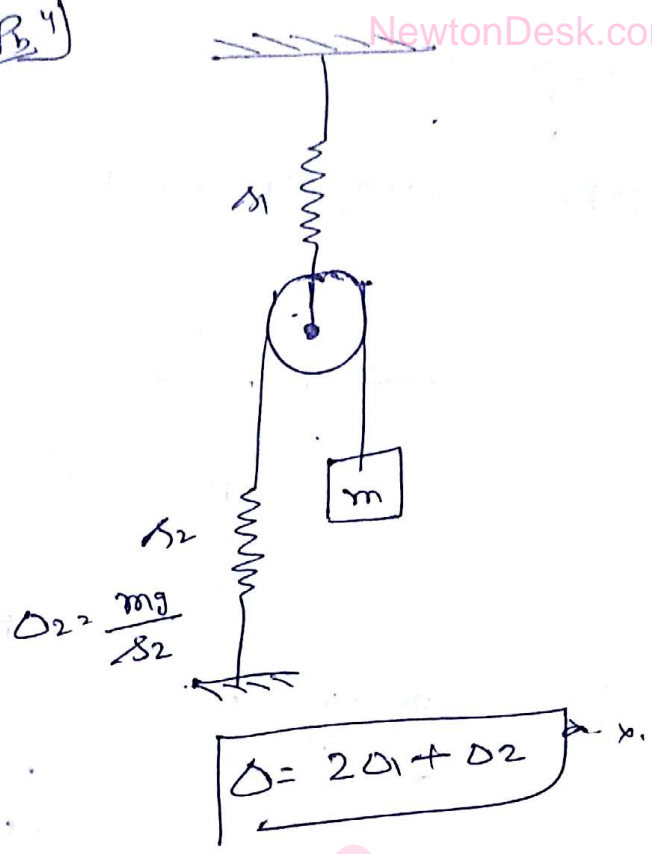
$$\Delta = \Delta_1 \frac{b}{a} + \Delta_2$$

will remain always same

B3)



B4)



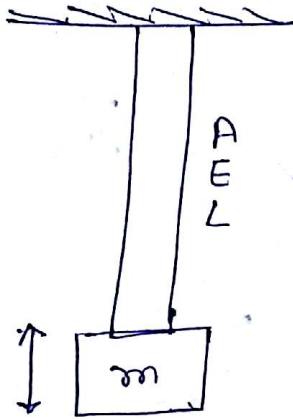
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Longitudinal vibrations :



Vibration along the length

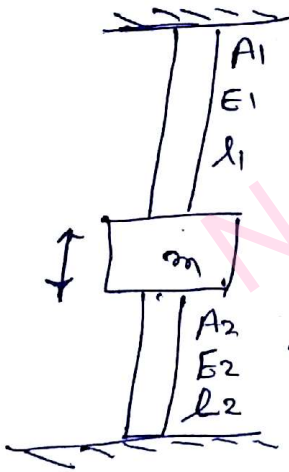


longitudinal stiffness

$$S = \frac{AE}{L}$$

$$\omega_n = \sqrt{\frac{S}{m}} \quad ***$$

Pb.



$$S_1 = \frac{A_1 E_1}{l_1}$$

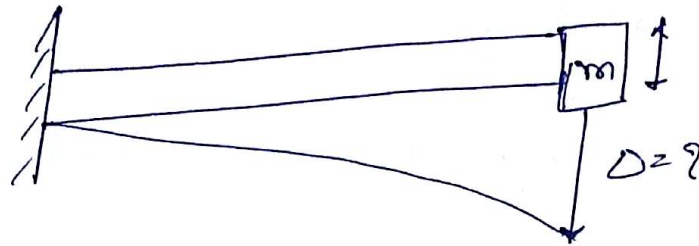
$$S_2 = \frac{A_2 E_2}{l_2}$$

$$\Delta S = S_1 + S_2$$

$$\omega_n = \sqrt{\frac{\Delta S}{m}}$$

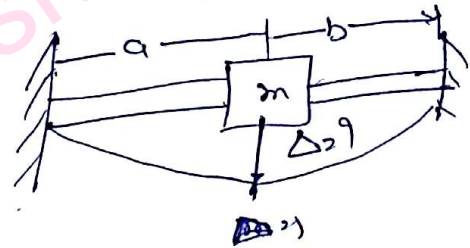
Transverse vibration of Beams

vibration in the dirⁿ \perp to length



$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{S}{m}}$$

stiffness of system
for Transverse vib



Damped System (Kinetic friction $\neq 0$)



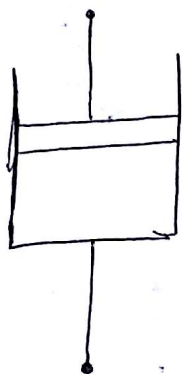
Technical Name of Kinetic friction in vibration system



Damping



And damping is represented by the symbol of



Damping in Any system

friction b/w dry surfaces

(Coulomb Damping)

Very high

Viscous Damping
(Very low)

Damping force in any system is proportional to \dot{x}

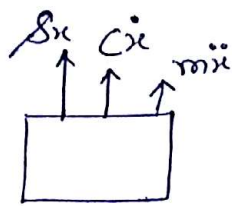
$$(\text{Damping force})_{\text{system}} \propto \dot{x}$$

$$= c \dot{x}$$

↳ Coefficient of Damping

At $t = t$

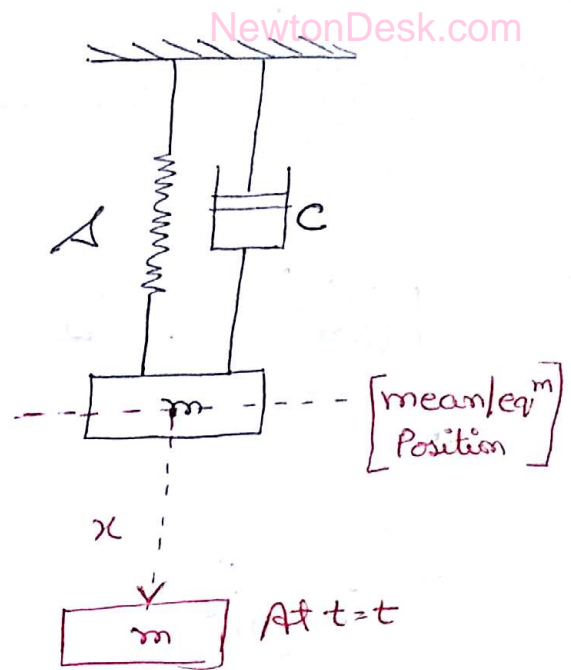
Free Body diagram of System



$$m\ddot{x} + C\dot{x} + Sx = 0$$

$$\ddot{x} + \left(\frac{C}{m}\right)\dot{x} + \omega_n^2 x = 0 \quad \text{--- (A)}$$

Equation of Damped System



And the solution of above eqnⁿ is

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad \alpha_1 \neq \alpha_2$$

or

$$x = (A + Bt)e^{\alpha t} \quad \longrightarrow \quad \boxed{\alpha_1 = \alpha_2 = \alpha}$$

$\alpha_{1,2} \rightarrow$ Roots of Auxiliary eqnⁿ

$$\boxed{\alpha^2 + \frac{C}{m}\alpha + \omega_n^2 = 0}$$

$$\alpha_{1,2} = -\frac{C}{m} \pm \sqrt{\left(\frac{C}{m}\right)^2 - 4\omega_n^2}$$

$$\alpha_{1,2} = -\frac{C}{2m} \pm \sqrt{\left(\frac{C}{2m}\right)^2 - \omega_n^2}$$

$$\frac{\left(\frac{C}{2m}\right)^2}{(\omega_n)^2} = \text{Degree of Dampness}$$

$$\sqrt{\frac{\left(\frac{C}{2m}\right)^2}{\omega_n^2}} = \zeta \Rightarrow \text{Damping factor} \\ \text{or} \\ \text{Damping Ratio}$$

$$\zeta = \sqrt{\frac{C^2}{4m^2} \cdot \frac{s}{m}} = \frac{C}{2\sqrt{sm}}$$

$$\boxed{2\zeta\omega_n} \rightarrow \zeta \times \frac{C}{2\sqrt{sm}} \times \sqrt{\frac{s}{m}} \Rightarrow \frac{C}{m}$$

$$\boxed{2\zeta\omega_n = \frac{C}{m}}$$

finally

equⁿ of Damped System

$$\Rightarrow \boxed{\ddot{x} + (2\zeta\omega_n)\dot{x} + (\omega_n^2)x = 0} \quad \text{***}$$

Solⁿ of this equⁿ,

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \quad (\alpha_1 \neq \alpha_2)$$

or

$$x = (A + Bt)e^{\alpha t}$$

where,

$$\alpha_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

If $\zeta > 1 \Rightarrow$ Over Damped System (Overdamping Coulomb damping)

If $\zeta = 1 \Rightarrow$ Critically Damped System (Critical damping)

If $\zeta < 1 \Rightarrow$ Under damped System (Under damping / viscous damping)

(i) if $\zeta > 1$ Over Damped System

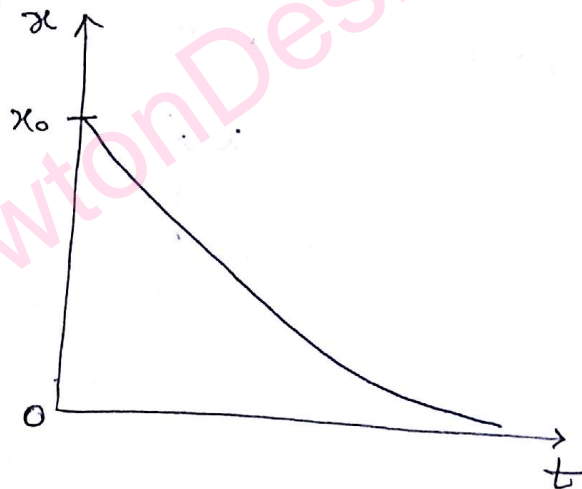
The solⁿ will be

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$x = Ae^{(-\zeta + \sqrt{\zeta^2 - 1})\omega_n t} + Be^{(-\zeta - \sqrt{\zeta^2 - 1})\omega_n t}$$

↓

No vibration in this system. e.g. Door closer



Over Damped System

ii) Critically Damped System ($\zeta = 1$)

$$\alpha_1 = \alpha_2 = \alpha = -\omega_n$$

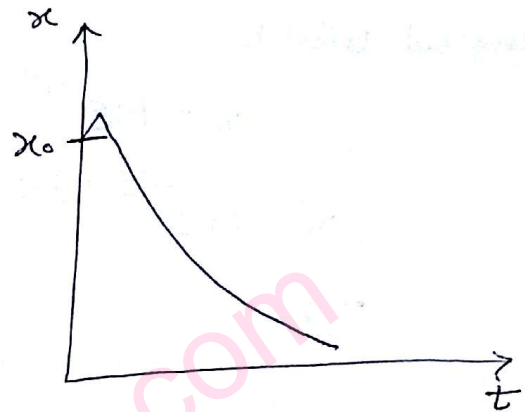
The solⁿ will be;

$$x = (A + Bt)e^{-\omega_n t}$$

Not visⁿ

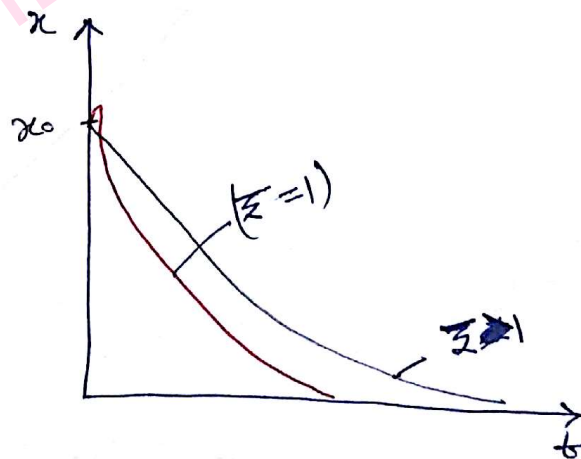
A, B are Constant

AK-47 example
660/min



Note

Critical Damping Response is much faster than over damping system.



iii) Under Damped System ($\zeta < 1$)

$$\alpha_{1,2} = -\zeta \omega_n \pm i \sqrt{1 - \zeta^2} \cdot \omega_n$$

$\omega_d = \text{Const.}$

$$\alpha_{1,2} = (-\zeta \omega_n \pm i \omega_d)$$

This solⁿ will be;

$$x = A e^{\alpha_1 t} + B e^{\alpha_2 t}$$

$$x = A e^{(-\zeta \omega_n + i \omega_d)t} + B e^{(-\zeta \omega_n - i \omega_d)t}$$

$$x = e^{-\zeta \omega_n t} \left[\underbrace{(A+B)}_{X \sin \phi} \cos \omega_d t + \underbrace{i(A-B)}_{X \cos \phi} \sin \omega_d t \right]$$

$$x = e^{-\zeta \omega_n t} X \sin \phi (\omega_d t + \phi)$$

$x = X e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$

$X, \phi \text{ Const}$

Amplitude
↓
↓ with time

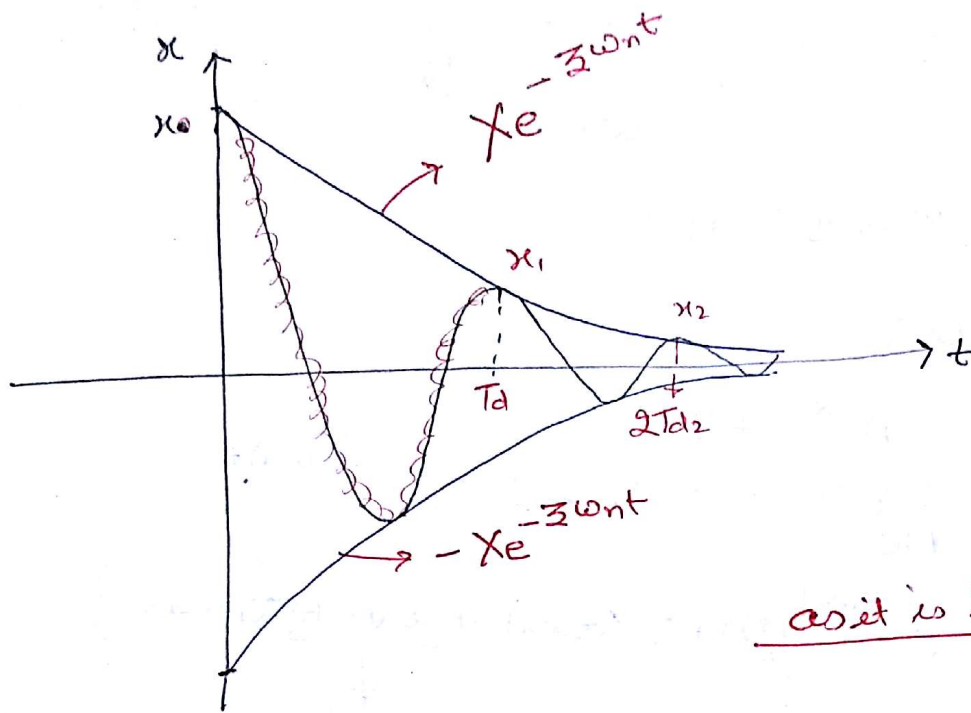
Vibⁿ with frequency

$\omega_d = \sqrt{1 - \zeta^2} \cdot \omega_n$

- Const

$$T_d = \frac{2\pi}{\omega_d} \quad \text{Rec - Const}$$

$$F_d = \frac{\omega_d}{2\pi} \quad \text{Hz - Const}$$



as it is harmonic motion

At $t=0$

$$x_0 = X \sin \phi$$

At $t=T_d$

$$x_1 = X e^{-z \omega_n (T_d)} \sin \phi$$

At $t=2T_d$

$$x_2 = X e^{-z \omega_n (2T_d)} \sin \phi$$

Decrement Ratio

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} \dots = e^{z \omega_n T_d} = \text{Const} \quad **$$

Logarithmic Decrement (δ) :

$$\begin{aligned}\delta &= \ln e^{\zeta \omega_n T_d} \\ &= \zeta \omega_n T_d \\ &= \zeta \omega_n \frac{2\pi}{\sqrt{1-\zeta^2} \omega_n}\end{aligned}$$

$$\delta = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}} \quad **$$

Critical Damping Coefficient (C_c) :-

$$\frac{\zeta \omega_n = \frac{C}{m}}{\omega_n = \frac{C_c}{m}} \Rightarrow \zeta = \frac{C}{C_c} = \frac{\text{Actual Damping Coeff.}}{\text{Critical Damping Coeff.}}$$

Notes:

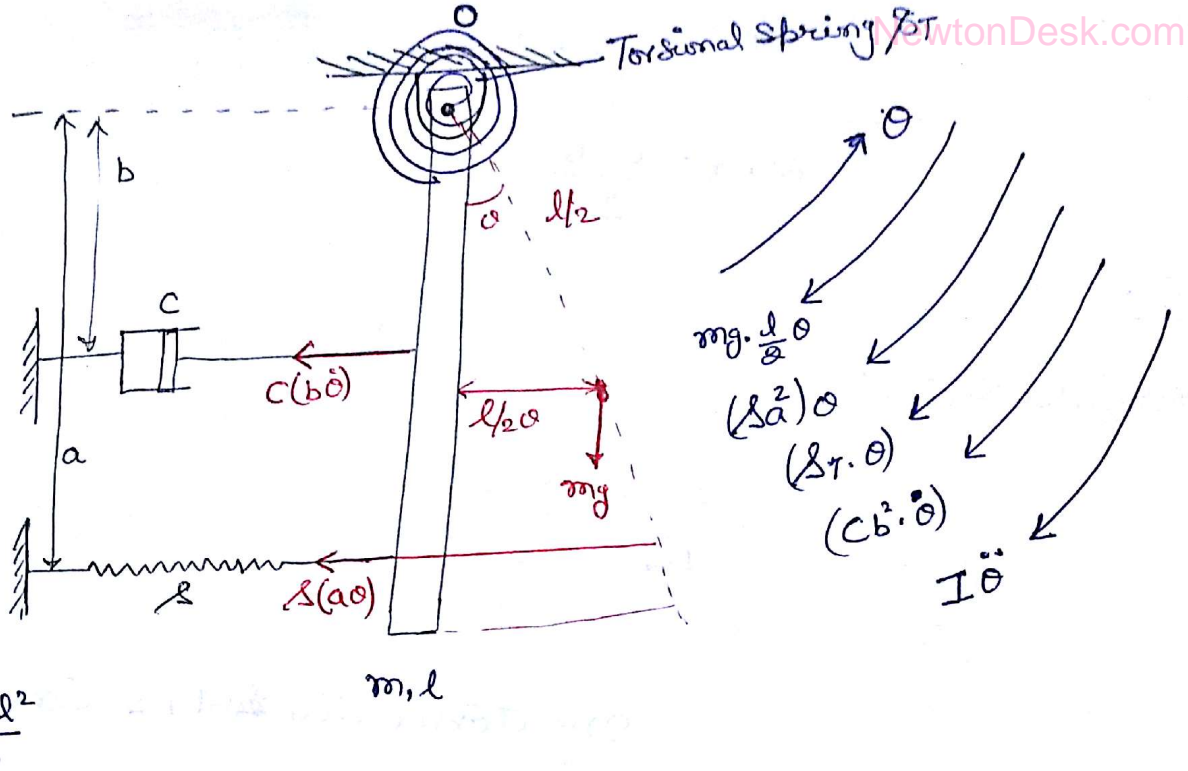
- The Ratio of Displacement of end of 3rd cycle to Start of 8th cycle is 2.5

$$\frac{x_3}{x_8} = 2.5$$

- The Ratio of Displacement of 3rd cycle to 8th cycle is 2.5

$$\frac{x_3}{x_8} = 2.5$$

Pb



$$I\ddot{\theta} + (Cb^2)\dot{\theta} + \left(\frac{mgl}{2} + \delta a^2 + \delta_T\right)\theta = 0$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

Problem

Q1) The Damping Coefficient in vibⁿ equⁿ will be $\rightarrow Cb^2$

Q2) $\omega_n \rightarrow \omega_n = \sqrt{\frac{\frac{mgl}{2} + \delta a^2 + \delta_T}{I}}$ $I = \frac{ml^2}{3}$

Q3) $\zeta = ? \rightarrow 2\zeta\omega_n = \frac{Cb^2}{I}$

Q4) $\omega_d = ?$

$$\omega_d = \sqrt{1 - \zeta^2} \cdot \omega_n$$

$\omega_n ?$

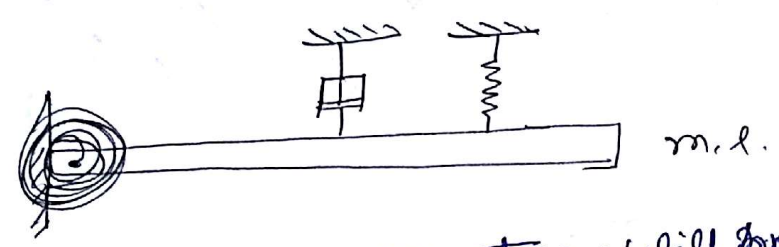
5

$C_c = ? \rightarrow$

$$2\omega_n = \frac{C_c b}{I}$$

Note:

Horizontal System

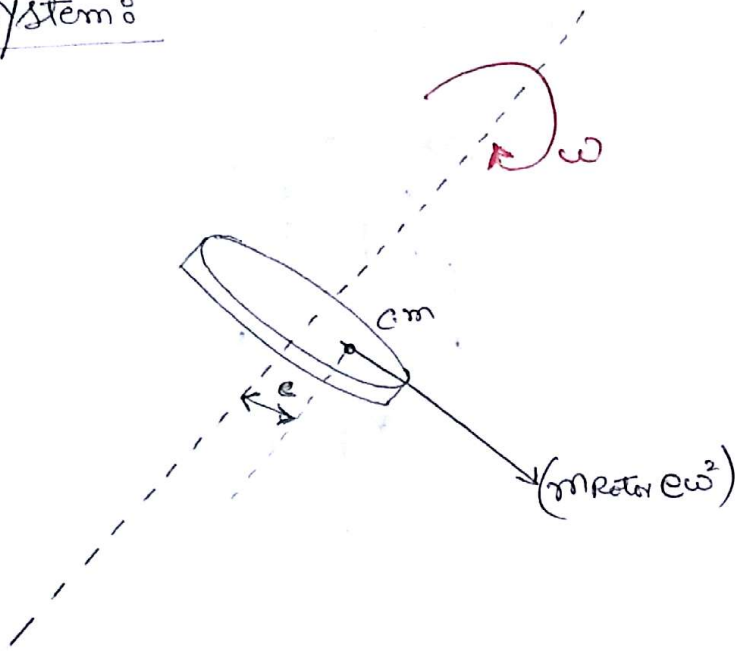


mg torque will ~~not~~ be considered

$$I\ddot{\theta} + (Cb^2)\dot{\theta} + (I\omega_n^2 + \beta t)\theta = 0$$

NewtonDesk.com

Vibrations Causing Unbalanced Forces in Rotating Mechanical System:



At $t = t$
 In a Particular dirⁿ
 $F_{un} = (M_{rotor} \cdot e \omega^2) \sin \theta$
 $= (M_{rotor} \cdot e \omega^2) \sin \omega t$

$$F_{un} = F_0 \sin \omega t$$

F_0 - Max. Value
 ω - Force Frequency.

in Case of Reciprocating unbalance:

At $t = t$

$$F_{un} = (M_{reci} \cdot e \omega^2) \sin \omega t$$

$$F_{un} = F_0 \sin \omega t$$

F_0 - max. Value
 ω - Force Frequency.

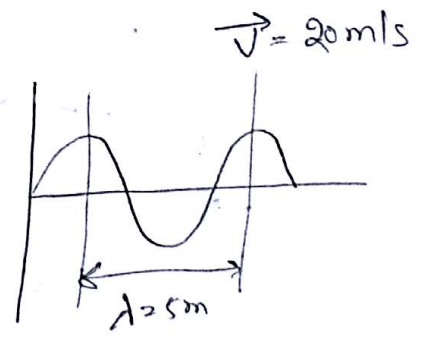
Wave form:

$$V = f \lambda$$

$$20 = f \times 5 \Rightarrow f = 4 \text{ Hz}$$

$$\omega = 2\pi f$$

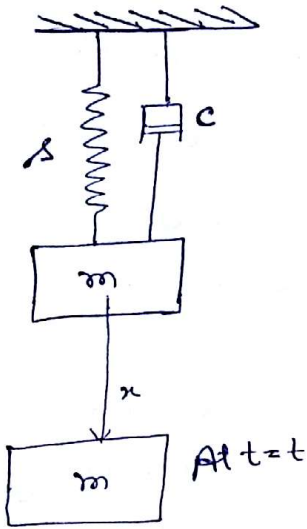
$$\omega = 2\pi \times 4 = 8\pi \text{ rad/s}$$



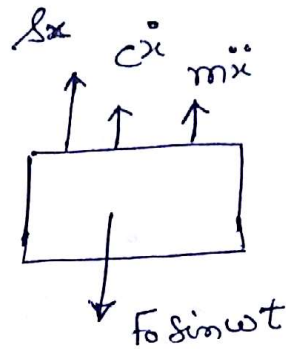
Direct form $\rightarrow F_{un} = 200 \cos(3)t$

\downarrow \downarrow
 F_0 ω

Forced Damped System :-



Freebody diagram at $t = t$



$$F_{em} = F_0 \sin \omega t$$

$F_0 \rightarrow$ Max. Value

$\omega \rightarrow$ Force frequency

$$m\ddot{x} + c\dot{x} + sx - F_0 \sin \omega t = 0$$

Equation of damped (Forced) system

$$\ddot{x} + (2\zeta \omega_n)\dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t$$

Solⁿ of this eqⁿ is;

$$x = C.F + P.I$$



After some time

$$C.F \rightarrow 0$$

$$x = P.I$$

P.I

D → Parameter
D² → -ω²

$$P.I = \frac{F_0 \sin \omega t}{D^2 + (2 \zeta \omega_n) D + \omega_n^2}$$

$$= \frac{F_0 \sin \omega t}{(\omega_n^2 - \omega^2) - (2 \zeta \omega_n) D} \left\{ \frac{(\omega_n^2 - \omega^2) - (2 \zeta \omega_n) D}{(\omega_n^2 - \omega^2) + (2 \zeta \omega_n) D} \right\} \frac{F_0 \sin \omega t}{(\omega_n^2 - \omega^2) - (2 \zeta \omega_n) D}$$

$$= \frac{F_0}{m} \frac{(\omega_n^2 - \omega^2) \sin \omega t - \frac{2 \zeta \omega \omega_n}{R \sin \phi} \cos \omega t}{(\omega_n^2 - \omega^2)^2 + (2 \zeta \omega \omega_n)^2}$$

$$= \frac{F_0}{m} \cdot \frac{R \sin(\omega t - \phi)}{R^2}$$

$$P.I = \frac{F_0/m}{(\omega_n^2 - \omega^2)^2 + (2 \zeta \omega \omega_n)^2} \sin(\omega t - \phi)$$

~~P.I = F_0/s~~

$$P.I = \frac{F_0/s}{\sqrt{\left(1 - \frac{\omega}{\omega_n}\right)^2 + \left\{\frac{2 \zeta \omega}{\omega_n}\right\}^2}} \sin(\omega t - \phi)$$

↓
Amplitude

↓
Vibration with Frequency ω

↓
Independent of time (more Dangerous)
New ends

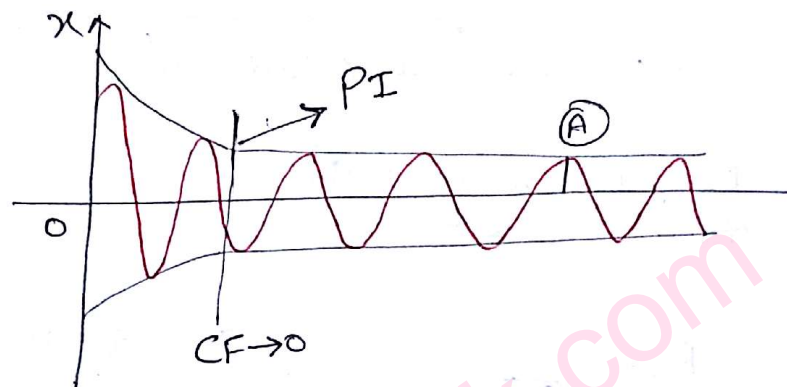
↓
After some time

CF → 0
 $P.I = X$

$$x = A \sin(\omega t - \phi)$$

where; $A = \frac{F_0/s}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$

Amplitude of steady ~~state~~ vibration (forced vibⁿ)
 doesn't depend on time



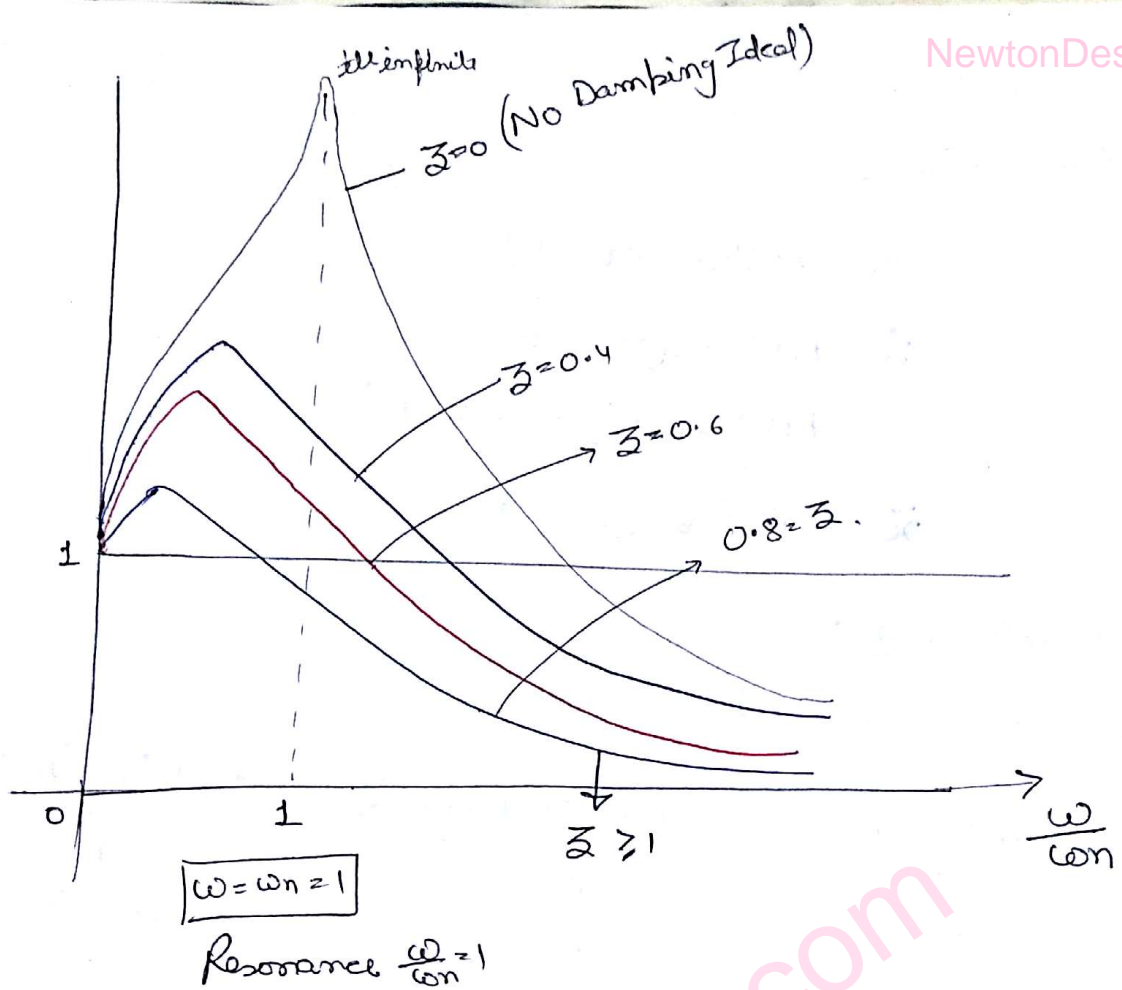
Vibration in Running Systems will never stop.
 Every mechanical system must have one running life

$M.F = \frac{A}{F_0/s}$

$$M.F = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

Magnification factor depends on :-

- 1) $\frac{\omega}{\omega_n}$
- 2) ζ



i) ↑ underdamping $\Rightarrow \zeta \downarrow$
 MF ↑ $\Rightarrow A \uparrow$ running life $\downarrow \downarrow$

ii) $(A)_{max}$ at $(MF)_{max}$
 at $\omega/\omega_n < 1 \rightarrow$ under damping (viscous)
 at $\omega/\omega_n = 1 \rightarrow$ No Damping
 at $\omega/\omega_n \geq 0 \rightarrow$ over/critical-damping (Coulomb).

3)
$$(A)_{Resonance} = \frac{(F_0/g)}{2\zeta} \propto \frac{1}{\zeta}$$

After some time,

$$x = A \sin(\omega t - \phi)$$

$$\dot{x} = A\omega \cos(\omega t - \phi)$$

$$= A\omega \sin\left(\frac{\pi}{2} + (\omega t - \phi)\right)$$

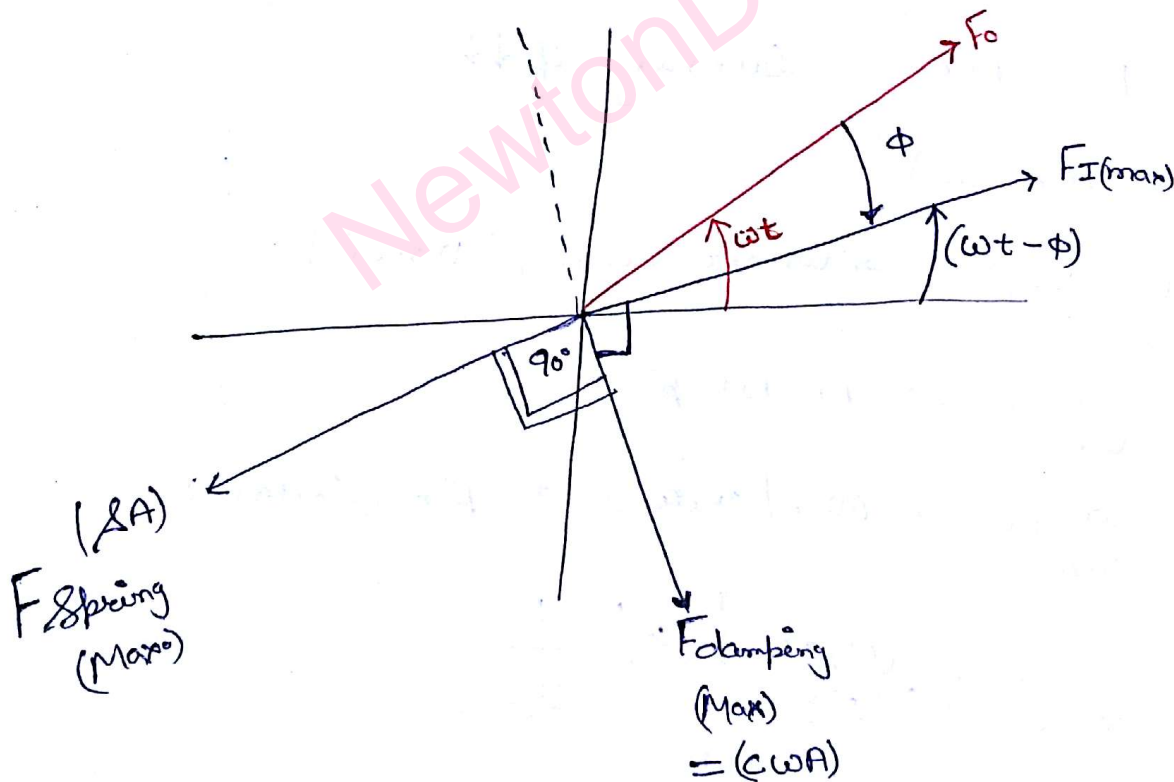
$$\ddot{x} = -A\omega^2 \sin(\omega t - \phi)$$

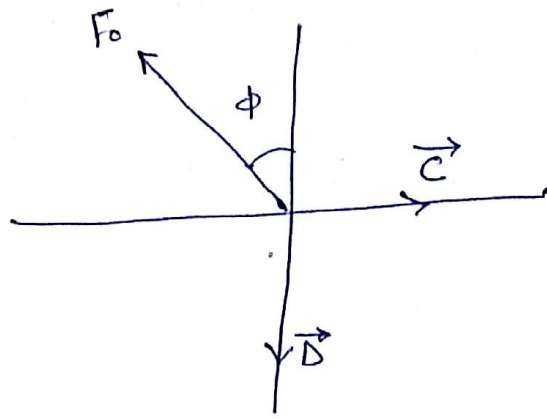
Basic equation was;

$$F_0 \sin \omega t - m\ddot{x} - c\dot{x} - \Delta x = 0$$

$$F_0 \sin \omega t + m\omega^2 A \sin(\omega t - \phi) - c\omega A \sin\left(\frac{\pi}{2} + (\omega t - \phi)\right) - \Delta A \sin(\omega t - \phi) = 0$$

\downarrow \downarrow \downarrow \downarrow
 $F_{\text{ext}} (\text{Max})$ $F_I (\text{max})$ $F_{\text{damping}} (\text{max})$ $(F_{\text{spring}} / \text{Elastic}) (\text{max})$





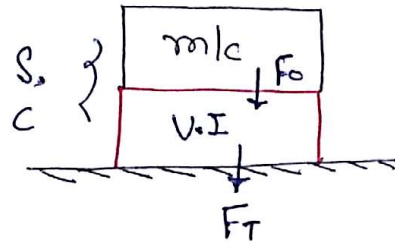
$\vec{C} = ?$ Damping force max.

$\vec{D} =$ Spring force max.

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Vibration Isolation :- (Foundation)

How to Isolate the ground from the vibration of Running Machine.



Where;

$F_T \rightarrow$ Transmitted force to ground. (ϵ)

$$F_T \lllll F_0 \quad *$$

$$\epsilon = \frac{F_T}{F_0}$$

$$0 < \epsilon < 1$$

$\epsilon \rightarrow 0$ Best

Transmissibility (ϵ) : (Performance of Vibration Isolation System)

$$F_T = \sqrt{(\beta A)^2 + (C\omega A)^2}$$

$$= \beta A \sqrt{1 + \left(\frac{C\omega A}{\beta A}\right)^2}$$

$$F_T = \beta A \sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

bcz both are \perp to each other.

F_s & $F_c \rightarrow$ More Response

$$F_0 = SA \sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}$$

$$\boxed{C = \frac{F_T}{F_0}}$$

$$\boxed{C = \frac{\sqrt{1 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}}}$$

$C \rightarrow$ depends on;

i) ω/ω_n

ii) ζ

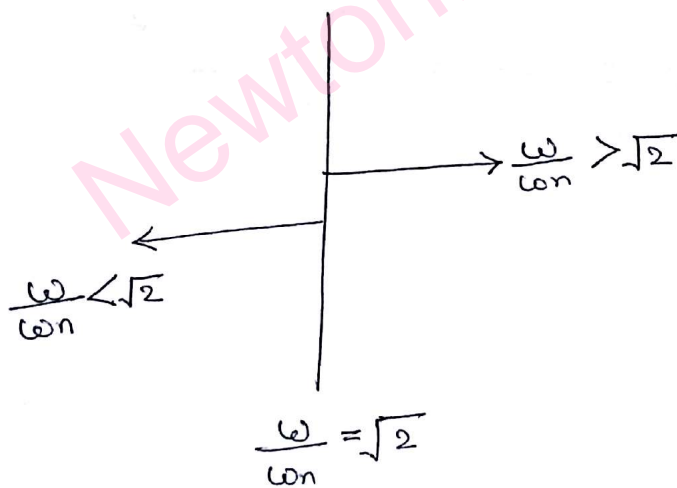
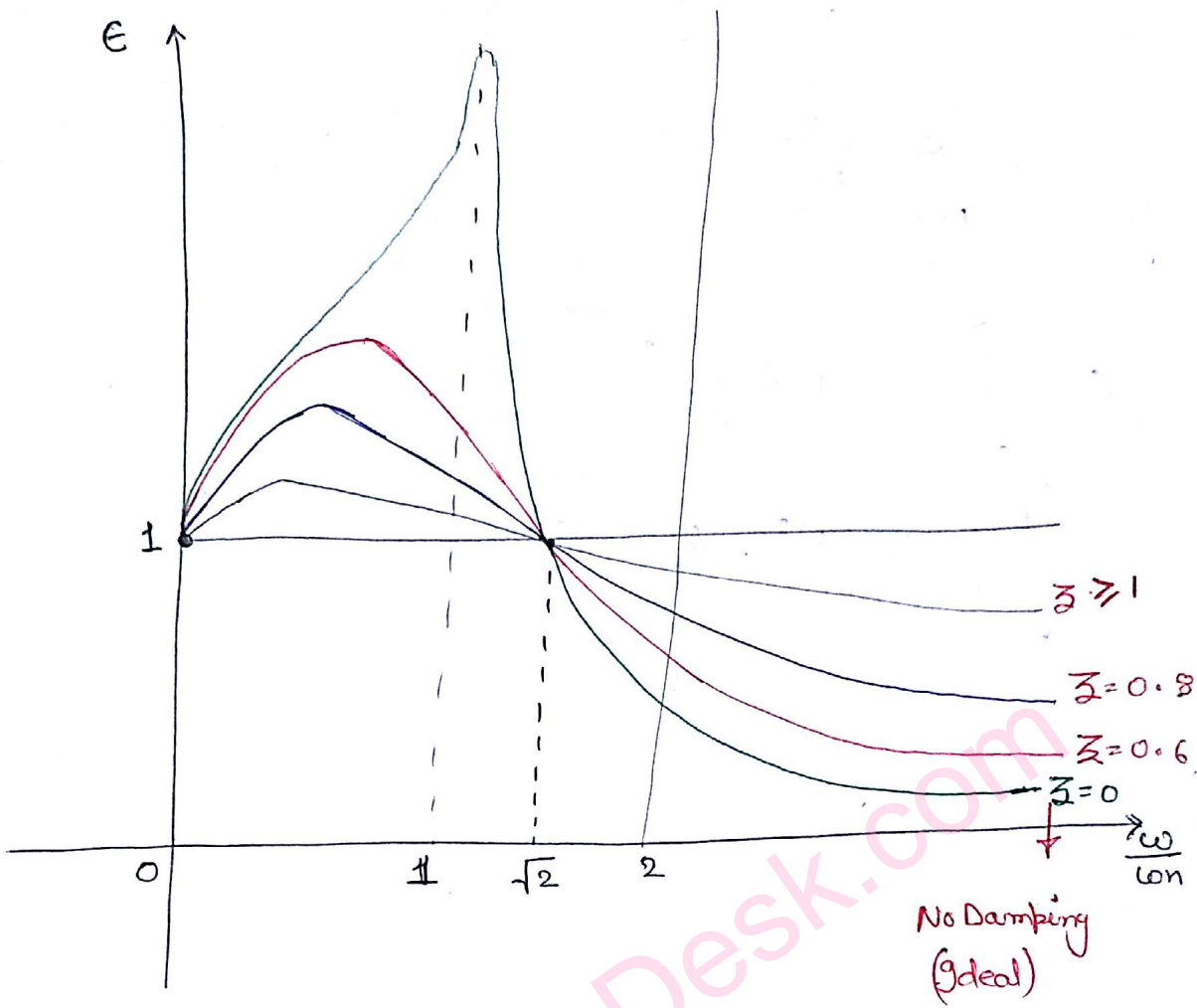
If,

$$\frac{\omega}{\omega_n} = 0$$

if

$$\frac{\omega}{\omega_n} = \sqrt{2}$$

$C = 1$ for all values of ζ *



i) If underdamping $\rightarrow \zeta < 1$

ζ will \uparrow if $\frac{\omega}{\omega_n} < \sqrt{2}$

ζ will \downarrow if $\frac{\omega}{\omega_n} > \sqrt{2}$

ζ will Remain same if $\frac{\omega}{\omega_n} = \sqrt{2}$

ii) Vibration Isolation will be effective,

when $\zeta < 1$

\downarrow
if $\left(\frac{\omega}{\omega_n} > \sqrt{2}\right)$

iii) if effective V.I Zone

$$\frac{\omega}{\omega_n} > \sqrt{2} \quad (\zeta < 1)$$

\rightarrow No damping is Best ($\zeta \rightarrow \infty$)

\rightarrow Damping becomes detrimental (harmful)

Pb) For effective Vibration Isolation the (ω_n)

i) ω ii) 2ω ~~iii) $\omega/4$~~ iv) $\cdot \text{claw}$

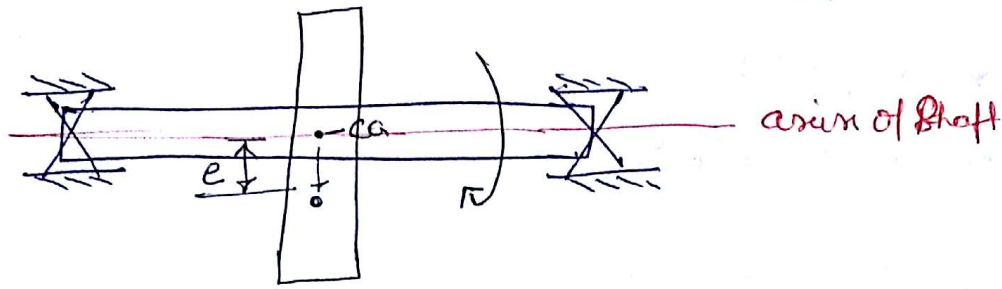
for effective V.I

$$\frac{\omega}{\omega_n} < \omega$$

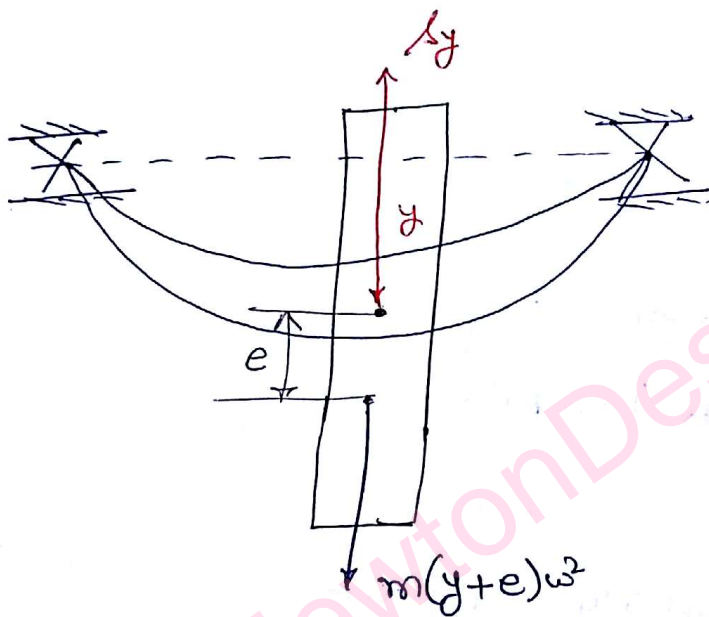
$$\frac{\omega}{\omega_n} > \sqrt{2}$$

$$\boxed{\omega_n < \frac{\omega}{\sqrt{2}}}$$

Whirling of Shaft :- (Shaft failure)



After some time



$$m(y+e)\omega^2 = \Delta y$$

$$m y \omega^2 + m e \omega^2 = \Delta y$$

$$\Delta y - m y \omega^2 = m e \omega^2$$

$$m y \omega^2 \left(\frac{\Delta}{m \omega^2} - 1 \right) = m e \omega^2$$

$$y = \frac{e}{\left(\frac{\omega}{\omega_n} \right)^2 - 1}$$

deflection in shaft.

In Some Systems

whose Running life is less
→ How to ↑ running life.

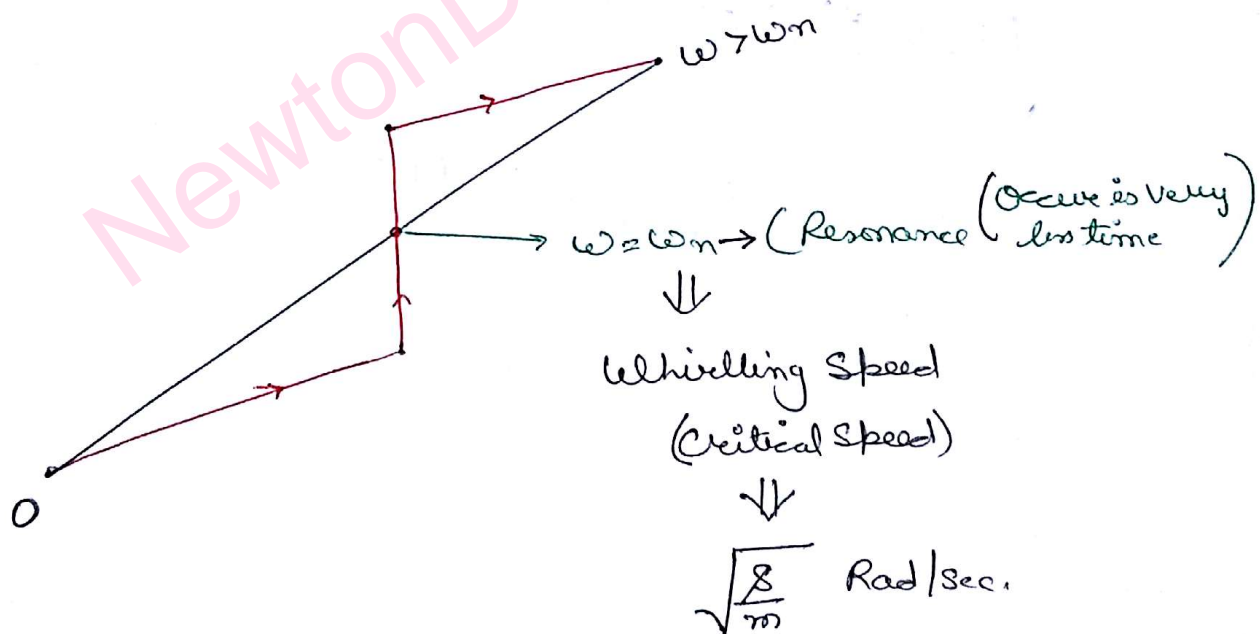
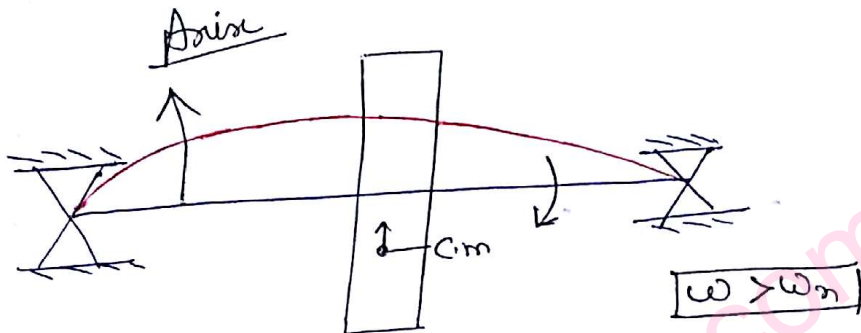
low life NewtonDesk.com

- 1) Heavy
- 2) more delicate item

If $\omega > \omega_n$

→ γ will become -ve.

→ Shaft bending will be in opposite dirⁿ.



(To stop opposite Torque is used)

Qm)

$$m = 17 \text{ Kg}$$

$$g = 1000 \frac{\text{N}}{\text{m}}$$

$$\omega_n = \sqrt{\frac{1000}{17}} \text{ Rad/s}$$

$$\frac{\omega}{\omega_n} = ?$$

$$\xi = 0.20$$

$$1) \quad A = \frac{F_0/s}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

$$2) \quad e = \frac{\sqrt{1 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}} \quad ?$$

$$e = \frac{F_T}{F_0} \dots F_T ?$$

$$m_{\text{Pesi}} = 2 \text{ Kg}$$

$$r = \frac{75}{2000} \text{ meter}$$

$$N = 500 \text{ rpm}$$

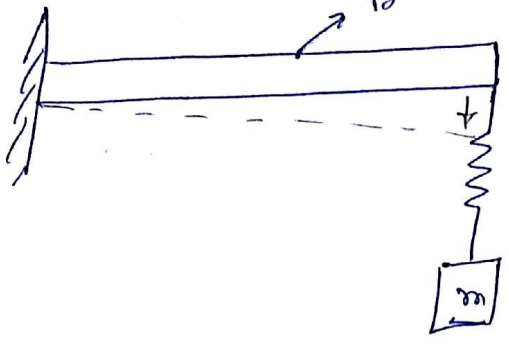
$$\omega = \frac{2\pi \times 500}{60} \text{ Rad/sec}$$

$$F_0 = 2 \times \frac{75}{2000} \times \frac{2\pi \times 500}{60} \text{ N}$$

Stiffness is opposite to deflection

P_b

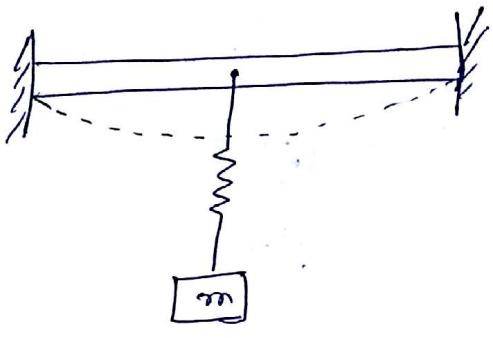
Q



|||



Q_2



|||



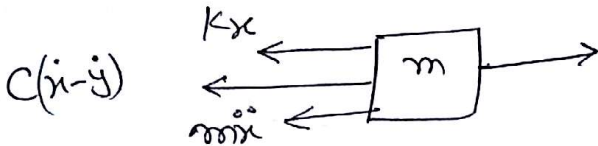
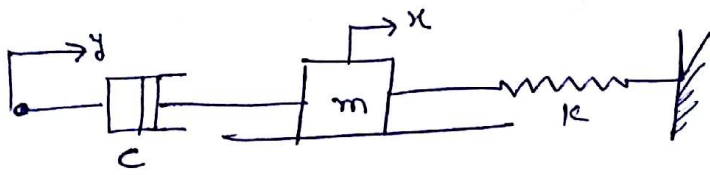
Q_3



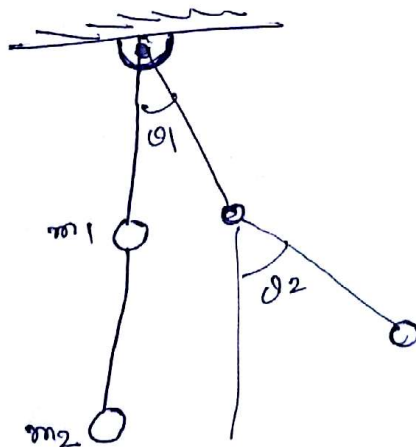
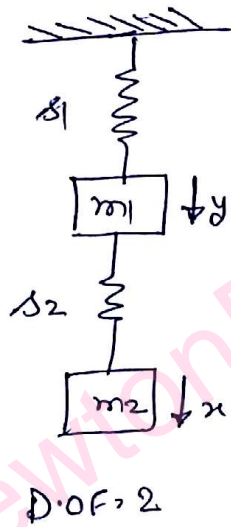
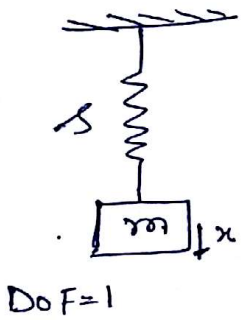
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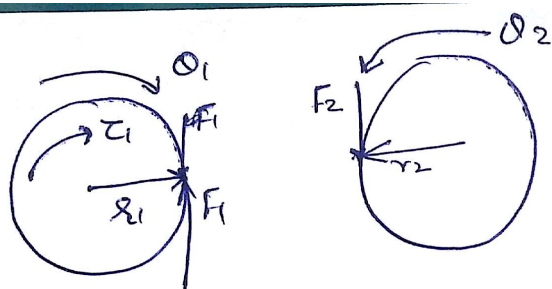


B



D.O.F of vibrating system





$$\begin{aligned} \tau_1 &= F_1 r_1 = I_1 \ddot{\theta}_1 \\ \tau_2 &= F_2 r_2 + I_1 \ddot{\theta}_1 \\ &= F_2 r_1 + I_1 \ddot{\theta}_1 \\ &= I_2 \left(\frac{r_1}{r_2}\right)^2 \ddot{\theta}_1 + I_1 \ddot{\theta}_1 \\ \Rightarrow \tau_1 &= \left[I_1 + I_2 \left(\frac{r_1}{r_2}\right)^2 \right] \ddot{\theta}_1 \end{aligned}$$

$$\begin{aligned} F_2 r_2 &= I_2 \ddot{\theta}_2 \\ F_2 &= \frac{I_2 \ddot{\theta}_2}{r_2} \\ r_1 \omega_1 &= r_2 \omega_2 \\ r_1 \dot{\alpha}_1 &= r_2 \dot{\alpha}_2 \\ r_1 \ddot{\theta}_1 &= r_2 \ddot{\theta}_2 \\ \ddot{\theta}_2 &= \frac{r_1}{r_2} \ddot{\theta}_1 \\ F_2 &= \frac{I_2 r_1}{r_2} \ddot{\theta}_1 \end{aligned}$$